Coxeter's rabbit

On p.13 of his "Introduction to Geometry", H.S.M. Coxeter invites the reader to see (and to use spontaneously) that with $s = (a+b+c)/2$, abc equals

(0) $s(s-b)(s-c) + s(s-c)(s-a) + s(s-a)(s-b) - (s-a)(s-b)(s-c)$

Proof $s(s-b)(s-c) + s(s-c)(s-a)$

$=$ {algebra}

$s(s-c)(2s-a-b)$

$=$ {definition of s}

(1) $s(s-c)c$

$s(s-a)(s-b) - (s-a)(s-b)(s-c)$

$=$ {algebra}

(2) $(s-a)(s-b)c$

Because both expressions (1) and (2) contain a factor c , so does (0); for reasons of symmetry, (0) also contains factors a and b , i.e. is a multiple of abc . The coëfficient equals 1 —as is trivially established with, say, $a,b,c := 2,2,2$ — and thus $abc = (0)$ has been proved. (End of Proof)

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