Geometric Modeling and Visualization

CS 384R, CAM 395T, BME 385J: Fall 2007

Take Home Final

Return answers before December 07, 2007, 11:59pm

Question 1: Consider a scalar function F defined on a two dimensional bounded domain D, and the level set curve family $Q : F = c$ for various $c \epsilon R^1$. Additionally, consider the region within domain D where $\nabla F \geq 0$ as exterior to the level set. For the twin cases of D being a triangle, and D being a square,

- (a) derive the length of Q as a function of c ,
- (b) derive the exterior and interior areas of Q as a function of c

Question 2. Consider a scalar function F defined on a three dimensional bounded domain D, and the level set surface family $Q : F = c$ for various $c \epsilon R^1$. Additionally, consider the region within domain D where $\nabla F \geq 0$ as exterior to the level set. For the twin cases of D being a tetrahedron, and D being a cube,

- (a) derive the surface area of Q as a function of c ,
- (b) derive the exterior and interior volumes of Q as a function of c

Question 3. Consider an arrangement of n^2 charged circular disks of radius r on a uniform rectilinear 2D n x n grid G with grid step size l. If q is the charge density per unit area of each disk, what is the total charge density of the arrangement as a function of r ? Note, that the topology/geometry of the union of the disks varies for discrete ranges of r for fixed l.

Question 4. Consider the vdW surface and the $L - R$ surface (definitions given in exercise 4) of a synthetic molecule M consisting of n^2 spherical atoms of equal radius r arranged uniformly on a rectilinear 2D n x n grid G with grid step size $l = 1.5 * r$ (a mono-layer sheet). For the $L - R$ surface, assume a solvent probe radius of w. What is the difference in the vdW and the $L - R$ surface areas of M as a function of n, r and w?

Question 5. For any point p on a smooth surface S in R^3, there is a well defined tangent plane T_p which is orthogonal to the normal vector n_p. For any vector θ on T_p the normal curvature $\kappa^n(\theta)$ is the curvature of the curve which is the intersection of the plane defined by n_p and θ and the surface S. Two principal curvatures of S at p κ_1 and κ_2 of S are the minimum and maximum values of all the normal curvatures at p. The mean curvature $\kappa_H = \frac{1}{2\pi}\int_0^{2\pi} \kappa^N(\theta) d\theta$ is expressible in terms of the principal curvatures, $\kappa^N(\theta) = (\kappa_1 \cos^2(\theta) + \kappa_2 \sin^2(\theta))$, and thereby yields $\kappa_H = \frac{(\kappa_1+\kappa_2)}{2}$. The Gaussian curvature is defined as $\kappa_G = \kappa_1\kappa_2$. Describe an algorithm or formula for estimating the principal curvatures, and hence the mean and Gaussian curvatures, for any point p on a quadratic A-patch, defined within a tetrahedron (i.e. barycentric Bernstein-Bezier basis).