Exercise 2: Algebraic Curve, Surface Splines - II

CS384R, CAM 395T, BME 385J: Fall 2007

September 11, 2007, Due: September 21, 2007

Question 1. Consider a simple polygon with $n \geq 3$ vertices. Describe a cubic A-spline construction scheme to smooth the polygon such that,

    (a) vertices of Q are C^1-interpolated (interpolatory spline)

    (b) vertices of Q are approximately interpolated to a user specified $\epsilon > 0$ (approximatory spline)

Question 2. Consider a spatial polygonal chain $Q^s \subseteq R^3$ with starting vertex $\overrightarrow{P_0}$ and ending vertex $\overrightarrow{P_n}$, $n \geq 3$. Construct a cubic A-spline, D, to C^1-interpolate the vertices of Q^s. Quantify the degrees of freedom of D that are still available to modify the spline without changing the topology of the polygonal chain Q^s.

Question 3. Consider a C^1-interpolatory quadratic A-spline, D, defined in the $x = 0$ plane (i.e., y-z plane) with none of the vertices $\overrightarrow{P_0}, \overrightarrow{P_1}, \ldots, \overrightarrow{P_n}$ incident on the z-axis.

    (a) Describe a square pyramidal A-patch data structure that represents the spline surface of revolution generated when D is revolved about the z-axis.

    (b) What is the degree of the spline surface?

    (c) What property of the A-spline would yield a lower degree spline surface of revolution?

    (d) Convert the square-pyramidal representation to a tetrahedral A-patch representation.

Question 4. Consider two circles in R^3, of radii 1 and 2, lying on the $x = 1$ and $y = 4$ planes, and with their centers on the x and y axis respectively.

    (a) Compute A-spline representation of each circle.

    (b) Compute a joining surface that interpolates the circles and contains the origin. Give your answer as an A-patch representation.

    (c) Is your solution the lowest degree algebraic surface and with the fewest number of A-patches ?

Question 5. The topology of an A-patch is the local topology of the level set of the algebraic function within its bounding linear finite element (simple polyhedron), and related to the critical points of the algebraic function within the finite element. Describe the possible topologies of (a) quadratic A-patch in a tetrahedron (b) tri-linear A-patch in a cube (c) combination [linear,quadratic] A-patch in a triangular prism