

LOGICAL IDENTITIES

$P \equiv P \wedge P$	idempotence of \wedge
$P \equiv P \vee P$	idempotence of \vee
$P \vee Q \equiv Q \vee P$	commutativity of \vee
$P \wedge Q \equiv Q \wedge P$	commutativity of \wedge
$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$	associativity of \vee
$(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$	associativity of \wedge
$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$ $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$	DeMorgan's Laws
$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$ $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$	distributivity of \wedge over \vee distributivity of \vee over \wedge
$P \vee T \equiv T$ $P \wedge F \equiv F$	domination laws
$P \wedge T \equiv P$ $P \vee F \equiv P$	identity laws
$P \vee \neg P \equiv T$ $P \wedge \neg P \equiv F$	negation laws
$\neg(\neg P) \equiv P$	double negation law
$P \vee (P \wedge Q) \equiv P$ $P \wedge (P \vee Q) \equiv P$	absorption laws
$P \rightarrow Q \equiv \neg P \vee Q$	implication
$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$	contrapositive
$P \leftrightarrow Q \equiv [(P \rightarrow Q) \wedge (Q \rightarrow P)]$	equivalence
$[(P \wedge Q) \rightarrow R] \equiv [P \rightarrow (Q \rightarrow R)]$	exportation

Rules of Inference

Rule	Corresponding Tautology	Name
$\frac{P \rightarrow Q}{P} \therefore Q$	$(P \rightarrow Q) \wedge P \rightarrow Q$	modus ponens
$\frac{P \rightarrow Q}{\neg Q} \therefore \neg P$	$(P \rightarrow Q) \wedge \neg Q \rightarrow \neg P$	modus tollens
$\frac{P \rightarrow Q}{Q \rightarrow R} \therefore P \rightarrow R$	$[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightarrow (P \rightarrow R)$	hypothetical syllogism
$\frac{P \vee Q}{\neg P} \therefore Q$	$[(P \vee Q) \wedge \neg P] \rightarrow Q$	disjunctive syllogism
$\frac{P}{\therefore P \vee Q}$	$P \rightarrow (P \vee Q)$	addition
$\frac{P \wedge Q}{\therefore P}$	$(P \wedge Q) \rightarrow P$	simplification
$\frac{P}{Q} \therefore P \wedge Q$	$[(P) \wedge (Q)] \rightarrow (P \wedge Q)$	conjunction
$\frac{P \vee Q}{\neg P \vee R} \therefore Q \vee R$	$[(P \vee Q) \wedge (\neg P \vee R)] \rightarrow (Q \vee R)$	resolution
$\frac{(P \rightarrow Q) \wedge (R \rightarrow S)}{P \vee R} \therefore Q \vee S$		constructive dilemma
$\frac{(P \rightarrow Q) \wedge (R \rightarrow S)}{\neg Q \vee \neg S} \therefore \neg P \vee \neg R$		destructive dilemma

Logical Relationships with Quantifiers

Law	Name
$\neg\forall xP(x) \equiv \exists x\neg P(x)$ $\neg\exists xP(x) \equiv \forall x\neg P(x)$	DeMorgan's laws for quantifiers
$\forall xP(x) \wedge \forall xQ(x) \equiv \forall x(P(x) \wedge Q(x))$ $\exists x(P(x) \vee Q(x)) \equiv \exists xP(x) \vee \exists xQ(x)$	distributivity of \forall over \wedge distributivity of \exists over \vee

Rules of Inference with Quantifiers

Rule of Universal Instantiation

$$\frac{\forall xP(x)}{\therefore P(c)} \text{ (where } c \text{ is some element of } P\text{'s domain)}$$

Universal Modus Ponens

$$\frac{\forall x(P(x) \rightarrow Q(x)) \quad P(c)}{\therefore Q(c)}$$

Universal Modus Tollens

$$\frac{\forall x(P(x) \rightarrow Q(x)) \quad \neg Q(c)}{\therefore \neg P(c)}$$

Universal Hypothetical Syllogism

$$\frac{\forall x(P(x) \rightarrow Q(x)) \quad \forall x(Q(x) \rightarrow R(x))}{\therefore \forall x(P(x) \rightarrow R(x))}$$

Universal Generalization:

$$\frac{P(c) \text{ for arbitrary } c \text{ in } U}{\therefore \forall xP(x)}$$

Existential Instantiation

$$\frac{\exists xP(x)}{\therefore P(c) \text{ for some } c}$$

Existential Generalization

$$\frac{P(c) \text{ for some element } c}{\therefore \exists xP(x)}$$