

CS313K
Fall 2006
Final Exam
December 15, 2006

Name:
EID:

- Answer all questions. Please give **clear** and **rigorous** answers. The logic you use in drawing conclusions and completing your answers is most important.
- Use extra paper to determine your solutions and then copy them neatly onto these sheets.
- If you include more than one answer to a question, and it is not clear to us which one you intended as your final answer, we will grade the first one and ignore the other answer.
- Do not use results proved in class or homework unless you prove them first.
- Make sure you clearly write your name and EID on this page.

GOOD LUCK

<i>Question</i>	<i>Score</i>	<i>Maximum</i>
1a		7
1b		7
2		10
3		5
4		10
5		15
6a		12
6b		5
7a		9
7b		4
7c		3
8a		8
8b		3
8c		2
Total		100

1. Define appropriate predicates and convert the following English statements into logical notation. Let the domain be the set of all people.

(a) Babies are illogical.

(b) Susan likes everyone who dislikes Joe.

2. Prove or disprove: For all real numbers x and y , there exists a real number z such that $x + z = y - z$.

3. Assume that A , B and C are sets such that $A \subseteq B^c$ and $C \subseteq B$. Use a Venn diagram to represent the relationship between A , B and C .

4. Prove: For all positive integers n , if $n > 2$, then $n^2 - 1$ is not prime.

Proof Type:

Assumptions/Givens for proof:

Goals for proof:

Proof:

5. Prove that $7|(2^{3n} - 1)$ for all positive integers n . (You must use mathematical induction to receive credit for this question).

6. Let $A = \{(i, j) | i, j \in \mathbb{N}\}$. Define relation R on A as follows:
 $((a, b), (c, d)) \in R \leftrightarrow ad = cb$.

(a) Prove that R is an equivalence relation.

(b) Use set-builder notation to define the equivalence class of $(1, 1) \in A$.

7. Let S be the relation on \mathbb{R} that is defined as follows:
 $xSy \leftrightarrow x - y$ is an non-negative even integer.

(a) Prove that (\mathbb{R}, S) is a poset.

(b) Is S a total order on \mathbb{R} ? Prove your answer.

(c) Are there any minimal elements for this poset? If so, what are they?

8. Define $f : \mathbb{N} \rightarrow P$, where P is the set of odd natural numbers, by $f(x) = 2x - 1$.

(a) Is f a bijection? Prove your answer.

(b) What is $f(\{1, 3, 4\})$?

(c) What is the preimage of 11?