

CS313K
Spring 2007
Midterm 1
March 1, 2007

Name:

EID:

TA's name and discussion time:

- Answer all questions. Please give **clear** and **rigorous** answers. The logic you use in drawing conclusions and completing your answers is most important.
- Use extra paper to determine your solutions and then copy them neatly onto these sheets.
- Make sure you clearly write your name and EID on this page.

GOOD LUCK

<i>Question</i>	<i>Score</i>	<i>Maximum</i>
1a		3
1b		4
1c		3
2		15
3a		5
3b		5
4a		6
4b		6
4c		6
5a		8
5b		15
6a		5
6b		4
6c		15
Total		100

2. Use an equivalence proof (**not** a truth table) to show that $[P \wedge (P \rightarrow Q)] \rightarrow Q$ is a tautology.

3. Translate the following statements into logical notation. Clearly define all propositions and predicates.

(a) The product of three negative integers is negative. (Universe is \mathbb{Z})

(b) Some student in this class grew up in the same town as some other student in this class. (Universe is the set of all students in this class)

4. True or false? Prove your answer. For each statement, $U = \mathbb{Z}$.

(a) $\exists n \forall m [nm = m]$

(b) $\exists n \exists m [n^2 + m^2 = 10]$

(c) $\forall n \exists m [n^2 < m]$

5. Consider the following argument:

All pet owners are cool.

Anyone who doesn't own a pet is lonely.

There is some person who is not lonely.

Therefore someone is cool.

(a) Re-write this argument in logical notation. Define your own predicates and universe.

(b) Is this argument valid? Prove your answer. If your answer is no, give a counterexample. If your answer is yes, give a step by step validity proof.

6. **Theorem:** For any integers a, b and c , if $a|b$ and $a|c$, then $a|(b + c)$.

Recall: For integers r and s , we say $r|s$ if $s = kr$ for some $k \in \mathbb{Z}$.

(a) Re-write the theorem in logical notation. Define the universe.

(b) State both the givens/assumptions and goal(s) for a direct proof of this theorem.

(c) Give a formal direct proof of the theorem.