

**CS313K**  
**Spring 2007**  
**Midterm 2**  
**April 4, 2007**

**Name:**

**EID:**

**TA name and discussion time:**

- Answer all questions. Please give **clear** and **rigorous** answers. The logic you use in drawing conclusions and completing your answers is most important.
- Use extra paper to determine your solutions and then copy them neatly onto these sheets.
- Make sure you clearly write your name and EID on this page.

**GOOD LUCK**

<i>Question</i>	<i>Score</i>	<i>Maximum</i>
1		12
2		12
3a		12
3b		12
4		12
5a		8
5b		8
5c		8
6		8
7a		7
7b		7
Total		100

1. Use an equivalence proof (not a truth table) to show that  $[\neg P \wedge (P \vee Q)] \rightarrow Q$  is a tautology.

2. Prove: If  $A \subseteq B$ , then  $C - B \subseteq C - A$  for every set  $C$ .

3. Prove or disprove:

(a) For any sets  $A, B$  and  $C$ , if  $A \cup B = A \cup C$ , then  $B = C$ .

(b) For any sets  $A, B$  and  $C$ ,  $(A - C) \cap (C - B) = \emptyset$ .

4. Prove: For all real numbers  $x > 2$ , there is a negative real number  $y$  such that  $x = \frac{2y}{y+1}$ .

5. Let  $A = \{a\}$ ,  $B = \{T, F\}$  and  $C = \{0, 1, \{2\}\}$ . Write down the specified power set or Cartesian product:

(a)  $2^C$

(b)  $B \times C$

(c)  $2^{A \times B}$

6. Let  $A$ ,  $B$  and  $C$  be sets in some domain  $U$ . Draw a Venn diagram that represents the set  $A - \overline{B \cup C}$ .

7. Define the set  $A_i = (i, \infty)$  for all  $i \in \mathbb{Z}^{\geq 0}$ . Define the following sets:

- $\cup_{i=0}^n A_i$ , where  $n \in \mathbb{Z}^+$

- $\cap_{i=0}^{\infty} A_i$