

CS341 Automata Theory

CFLs Review Sheet

1. For each of the following languages, state whether the language is (I) regular, (II) context-free but not regular, or (III) not context-free. Prove your answer.

(a) $\{0^n 0^{2n} 0^{3n} | n \geq 0\}$

regular. The language is described by the regular expression $(000000)^*$.

(b) $\{xwx^R | x, w \in \{0, 1\}^+\}$

regular. $0(0 \cup 1)^+ 0 \cup 1(0 \cup 1)^+ 1$

(c) $\{xyz | x, y, z \in \{0, 1\}^*, |x| = |z| \text{ and } \#_0(x) \geq \#_0(z)\}$

This was an old homework question. regular: $(0 \cup 1)^*$.

(d) $L = \{1^{n^2} | n \geq 0\}$

not context-free.

Assume BWOC that L is context-free and let p be the pumping length given in the pumping lemma. Choose $w = 1^{p^2}$. Then $w \in L$ and $|w| \geq p$. So the Pumping Lemma says that there is some way to write $w = uvxyz$ so that $|vxy| \leq p$, $|vy| > 0$, and $uv^i xy^i z \in L$ for all $i \geq 0$.

Note that $0 < |vy| \leq p$. So

$uv^2 xy^2 z = 1^{p^2+r}$, where $r = |vy|$, and so $|uv^2 xy^2 z| = p^2 + r$. Then

$$p^2 < p^2 + r \leq p^2 + p < p^2 + 2p + 1 = (p + 1)^2.$$

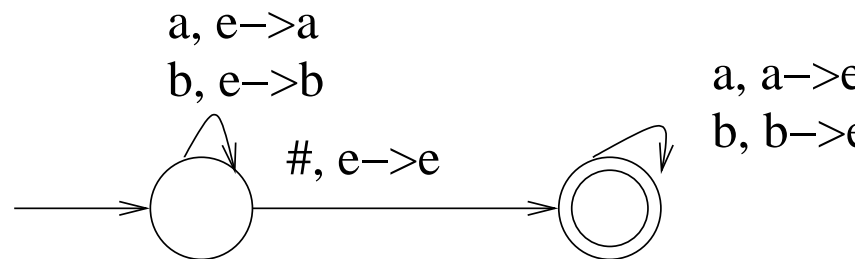
So the length of $uv^2 xy^2 z$ is strictly between two consecutive perfect squares, so $uv^2 xy^2 z \notin L$. \square

(e) $L(11(01 \cup 0)^*(01 \cup 0) \cup 10)$

regular.

(f) $\{w\#w^R | w \in \{a, b\}^*\}$

context-free, not regular.



Apply the regular language pumping lemma to string $w = 0^p 1^p \# 1^p 0^p$.

(g) $L = \{w | w \in \{a, b\}^*, w \neq w^R\}$

context-free, not regular.

CFG: $S \rightarrow aSa | bSb | T$

$T \rightarrow aUb | bUa$

$U \rightarrow aU | bU | \epsilon$

not regular: Show L^c is not regular (similar to proof in part f)). Since the regular languages are closed under complement, use a proof by contradiction to show L is not regular as well.

(h) $\{a^i b^j c^k | i < j \rightarrow k < j\}$

By the logical equivalence $p \rightarrow q \equiv \neg p \vee q$, we have

$$L = \{a^i b^j c^k | i < j \rightarrow k < j\}$$

$$= \{a^i b^j c^k | \neg(i < j) \vee (k < j)\} = \{a^i b^j c^k | i \geq j \geq 0\} \cup \{a^i b^j c^k | 0 \leq k < j\}.$$

So the CFL for L is

$$S \rightarrow S_0 | S_1$$

$$S_0 \rightarrow S_{01} C$$

$$S_{01} \rightarrow a S_{01} b | a S_{01} | \varepsilon$$

$$C \rightarrow c C | \varepsilon$$

$$S_1 \rightarrow A S_{11}$$

$$A \rightarrow a A | \varepsilon$$

$$S_{11} \rightarrow b S_{11} c | b S_{11} | \varepsilon$$

To see that L is not regular, let $w = b^{p+1}c^p$, where p is the pumping length in the pumping lemma. Let $w = xyz$, where $x = b^r$, $y = b^s$, and $z = b^{p+1-r-s}c^p$, where $0 < s \leq p$. We then have $xz = b^{p+1-s}c^p$ after pumping down. Since neither $0 \geq p + 1 - s$ nor $p < p + 1 - s$ is true, $xz \notin L$. Contradiction.

2. Prove that the context-free languages are closed under union and concatenation. (Don't copy your class notes - this should be good practice for proving a closure property on the exam).
3. Suppose L_1, L_2 are context-free languages. Is it necessarily true that $L_1 - L_2$ is context-free? Prove your answer.
No. We proved this in class.

4. Convert the following grammar to Chomsky Normal Form. $S \rightarrow A|B$

$$A \rightarrow aA | aC$$

$$B \rightarrow bB | bC$$

$$C \rightarrow pqrW$$

$$W \rightarrow TV$$

$$T \rightarrow t | \varepsilon$$

5. Show that the following CFG is ambiguous:

$$E \rightarrow E + E | E - E | E x E | (E) | a | b.$$

Two leftmost derivations of $axb+a$:

$$E \Rightarrow E + E \Rightarrow E x E + E \Rightarrow a x E + E \Rightarrow a x b + E \Rightarrow a x b + a$$

$E \Rightarrow ExE \Rightarrow axE \Rightarrow axE + E \Rightarrow axb + E \Rightarrow axb + a$

Since a string has two different leftmost derivations in the CFG, the CFG is ambiguous.

6. For CFG G and string w , give an algorithm to decide if $w \in L(G)$.

We did this in class.

7. Define a PDA that recognizes $\{0^n w 1^m \mid w \in \{0, 1\}^*, \#_0(w) = \#_1(w) \text{ and } m \geq n\}$

