CS341 Automata Theory
Homework Assignment ♯1

Do not forget to write your name and EID at the top of the first page of your solution set. No collaboration is allowed; all solutions submitted must be your own work. Solution sets must be stapled together.

1. For each of the languages below, describe them very precisely and succintly in English. In addition to your precise English definition, list 8 strings that are in the language.

   (a) $\{xyz | x, y, z \in \{a, b\}^*, |x| = |z| \text{ and } \#_a(x) \geq \#_a(z)\}$

   (b) $\{a_1 a_2 ... a_{2n} \in \{0, 1\}^* | n \geq 1, a_1 a_2 ... a_n \in L, a_i \in L \text{ for all } i\}$. Describe this language assuming that $L = \{0^n | n \geq 0\}$

   (c) $\{xax^R | x \in \{a, b, c\}^*\}$.

2. Define a relation R on $\mathbb{N} \times \mathbb{N}$ (where $\mathbb{N}$ is the set of natural numbers) by $R = \{(x, y) | x \in \mathbb{N}, y \in \mathbb{N} \text{ and x+y is even}\}$. Prove or disprove that R is an equivalence relation.

3. Write each of the following explicitly.

   (a) $\{1\} \times \{1, 2\} \times \{1, 2, 3\}$.

   (b) $\emptyset \times \{1, 2\}$.

   (c) $2^{\{1,2\}} \times \{1, 2\}$.

4. For each of the following statements, state whether it is TRUE or FALSE. Prove your answer.

   (a) $L_1 = L_2$ if and only if ${L_1}^* = {L_2}^*$.

   (b) $(\emptyset \cup \emptyset^*) \cap (\overline{\emptyset} - (\emptyset\emptyset^*)) = \emptyset$.

   (c) Every infinite language is the complement of a finite language.

5. Let C be a set of sets defined as follows.

   (i) $\emptyset \in C$.
   (ii) If $S_1 \in C$ and $S_2 \in C$, then $\{S_1, S_2\} \in C$.
   (iii) If $S_1 \in C$ and $S_2 \in C$, then $S_1 \times S_2 \in C$.
   (iv) Nothing is in C except that which follows from (i), (ii), and (iii).

   (a) Explain carefully why it is a consequence of (i) through (iv) that $\{\emptyset, \{\emptyset\}\} \in C$.

   (b) Give an example of a set S of ordered pairs such that $S \in C$ and $|S| > 1$.

6. Show each of the following.

   (a) $\{\varepsilon\}^* = \{\varepsilon\}$.

   (b) For any language L, $\emptyset L = L\emptyset = \emptyset$.

   (c) $\emptyset^* = \{\varepsilon\}$.

   (d) $L^* = L^+$ if and only if $\varepsilon \in L$.

7. Are the following sets closed under the given operations? Explain.

   (a) The negative integers under multiplication.

   (b) The positive integers under division.

   (c) The set of all strings over Σ under concatenation.

8. Give examples to show that the intersection of two countably infinite sets can be either finite or countably infinite, and that the intersection of two uncountable sets can be finite, countably infinite, or uncountable.

9. Prove that given any sets A, B, and C, $A - (B \cup C) = (A - B) \cap (A - C)$.

10. Show that the set of real numbers between 0 and 1 (inclusive) is uncountable.

11. Prove: The difference of an even number and an odd number is odd.

12. Prove or disprove: If $x \not| (m - n)$ then $x \not| m$ or $x \not| n$.