CS341 Automata Theory (Summer 2008)
Homework Assignment $\sharp 4$

Do not forget to write your name and EID.

1. Define the language that is generated by each of the following context-free grammars:
   (a) $S \rightarrow aSa|bSb|a|b$.
   (b) $S \rightarrow aS|bS|\varepsilon$.
   (c) $S \rightarrow aS|Sb|\varepsilon$.

2. For each of the following languages L, construct a context-free grammar that generates it:
   (a) $\{a^n b^m | m \geq n, m - n \ is \ even\}$.
   (b) $\{xc^n | x \in \{a,b\}^*, \sharp_a(x) = n \ or \ \sharp_b(x) = n\}$.
   (c) $\{xc^n | x \in \{a,b\}^*, \sharp_a(x) + \sharp_b(x) \geq n\}$.
   (d) $\{a^i b^j | 2i = 3j + 1\}$.

3. Construct pushdown automata that accept each of the following languages:
   (a) $L = \{a^m b^n |\ m \leq n \leq 2m\}$.
   (b) $L = \{w \in \{a,b\}^* |\ w$ has twice as many a's as b's$\}$.
   (c) $L = \{a^m b^n |\ m \geq n\}$.

4. For each of the following languages, each one is either (I) regular, (II) context-free but not regular, or (III) not context-free. Decide to which category each language belongs and prove your answer.
   (a) $L = \{a^i b^n | i, n \neq 0,\ i = n \ or \ i = 2n\}$.
   (b) $L = \{xy | x, y \in \{a,b\}^*,\ |x| = |y|\}$.
   (c) $L = \{0^i 1^j | j = i^2\}$
   (d) $L = \{x_1 \# x_2 \# x_3 ... \# x_k | k \geq 2, x_i \in \{a,b\}^* \forall i$, and $x_i = x_j$ for some $i \neq j\}$
   (e) $L = \{0^n \# 0^{2n} \# 0^{3n} | n \geq 0\}$

5. Consider $L = \{0^i 1^i | i \geq 0\}$. What is wrong with the following proof that L is not regular? Describe the problem clearly, and correct the proof.
   Assume BWOC that L is regular, and let p be the pumping length. Choose $w = 0^p 1^p$. So $w \in L$ and $|w| \geq p$. So the pumping lemma tells us that $w = xyz$ such that $|xy| \leq p$, $|y| > 0$, and $xy^i z \in L$ for all $i \geq 0$.

Since $|xy| \leq p$, it follows that
$x = 0^i, y = 0^j, z = 0^{p-i-j}1^p$ where $j > 0$.
Let j = 1. Then $xz = xy^0z = 0^{p-1}1^p$, and so the $\#_0(xz) \neq \#_1(xz)$. So $xz \notin L$. Contradiction! So L is not regular.