CS341 Automata Theory (Summer 2008)
Homework Assignment ♯5

Do not forget to write your name and EID.

1. Consider this CFG G:
   $S \rightarrow D|E$
   $D \rightarrow dD|dDe|dF$
   $E \rightarrow Ec|dEe|Fe$
   $F \rightarrow \varepsilon$

   (a) Give a concise definition of L(G).
   (b) Show the leftmost derivation of string ddeee in L(G).
   (c) Rewrite G in Chomsky Normal Form. Use the algorithm given in class, and show each step.

2. Define a PDA that recognizes $L = \{a^m b^n | m, n \geq 0, n = 3m \text{ or } n = 2m\}$.

3. Define a CFG that generates $L = \{w \in \{0,1\}^* | w \text{ contains at least as many 0s as 1s}\}$.

4. For each language indicate whether the language is (I) regular, (II) context-free but not regular, (III) semi-decidable but not context-free. Prove your answer. For these languages, if you choose (III), draw a state diagram of the Turing machine that recognizes the language (instead of giving an English description of the TM).

   (a) $L = \{w \in \{a,b\}^* | \#_0(w) = \#_1(w)$, and at any point in string w, there are not more 1s than 0s before that point$\}$
   (b) $L = \{a^{3k} b^{2k} c^k | k \geq 0\}$

5. Indicate whether each of the following statements are true or false, and prove your answer.

   (a) If $L_1, L_2, L_3, ...$ are context-free languages, then $L = \bigcup L_i$ is context-free.
   (b) If L_1, L_2 are languages such that $L_1 \cap L_2$ is regular, then L_1 and L_2 are context-free.

6. (a) Design a Turing machine M that recognizes $\{0^{2^n} | n \geq 0\}$.
   (b) Give the sequence of configurations that M enters when started on the indicated input string:
   i. 0
   ii. 0000

7. Examine the formal definition of a Turing machine that we gave in class to answer the following questions, and explain your reasoning.

(a) Can a TM ever write the blank symbol on its tape?

(b) Can the tape alphabet be the same as the input alphabet?

(c) Can a TM's read head ever be in the same location before and after a transition?

(d) Can a TM have only one state?

8. Give state diagrams for TMs that recognize the following languages:

   (a) $\{w \in \{0,1\}^* | w \text{ contains twice as many 0s as 1s}\}$

   (b) $\{0^k 1 0^k 1 0^k | k \geq 0\}$ (Define a decider for this language).