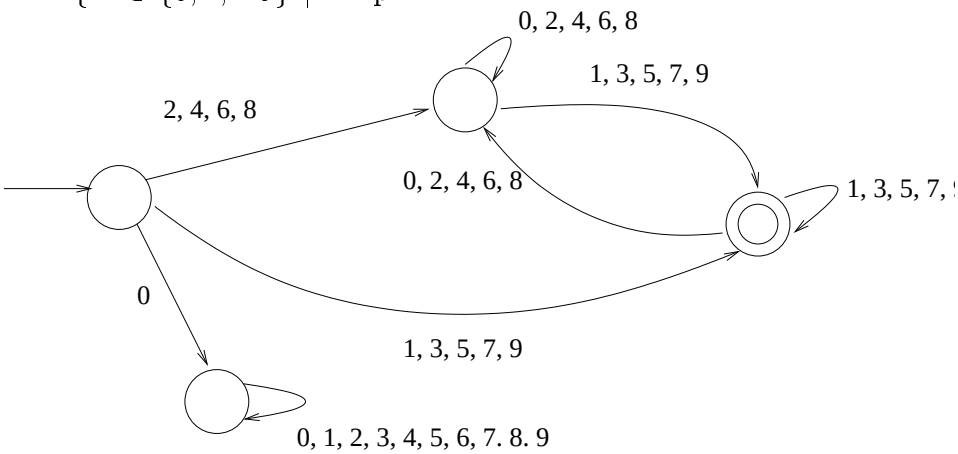
CS341 Automata Theory

Summer 2005

Midterm Practice Problems - Selected Solutions

1. Define a deterministic finite automaton for the following languages:

   (a) $L = \{w \in \{0, 1, ...9\}^* | w$ represents an odd natural number with no leading 0s$\}$



   (b) $L = \{w \in \{0, 1, 2\}^* | w$ does not end in 12$\}$

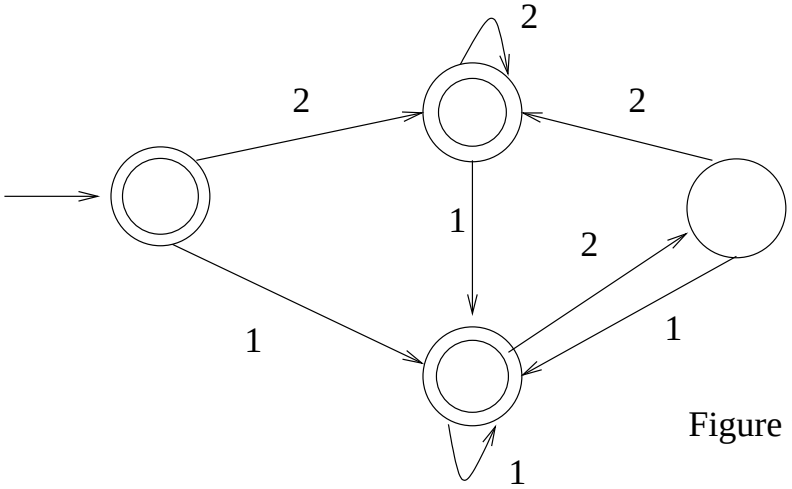
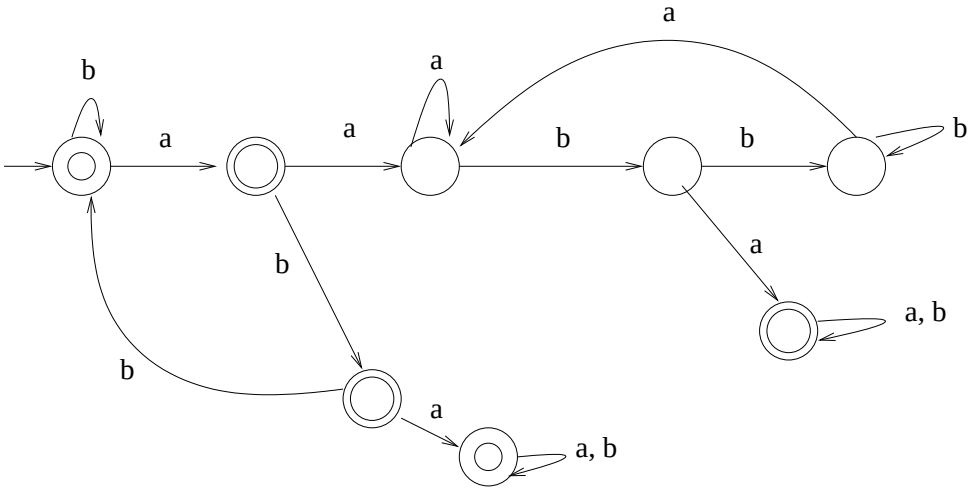


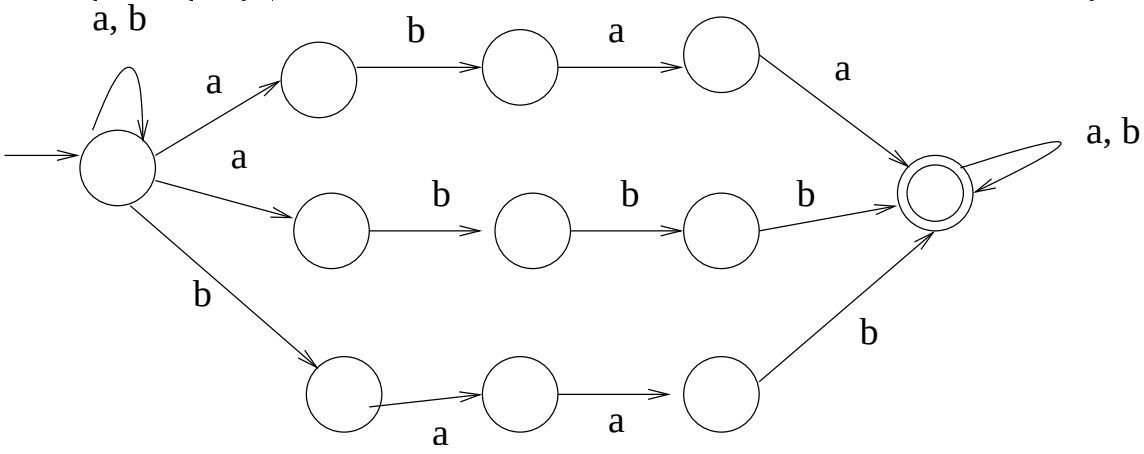
Figure for 1b

   (c) $L = \{w \in \{a, b\}^* | w$ contains the string aba or w does not contain aa$\}$



2. Define an NFA that recognizes the following languages:

   (a) $L = \{w \in \{a,b\}^* | w$ contains at least one instance of abaa, abbb, or baab$\}$



   (b) $L = \{a^n b a^m | n, m \geq 0, n \equiv_3 m\}$

   (c) $L = \{w \in \{a,b,c\}^* | w$ contains substring abb and substring bbc$\}$

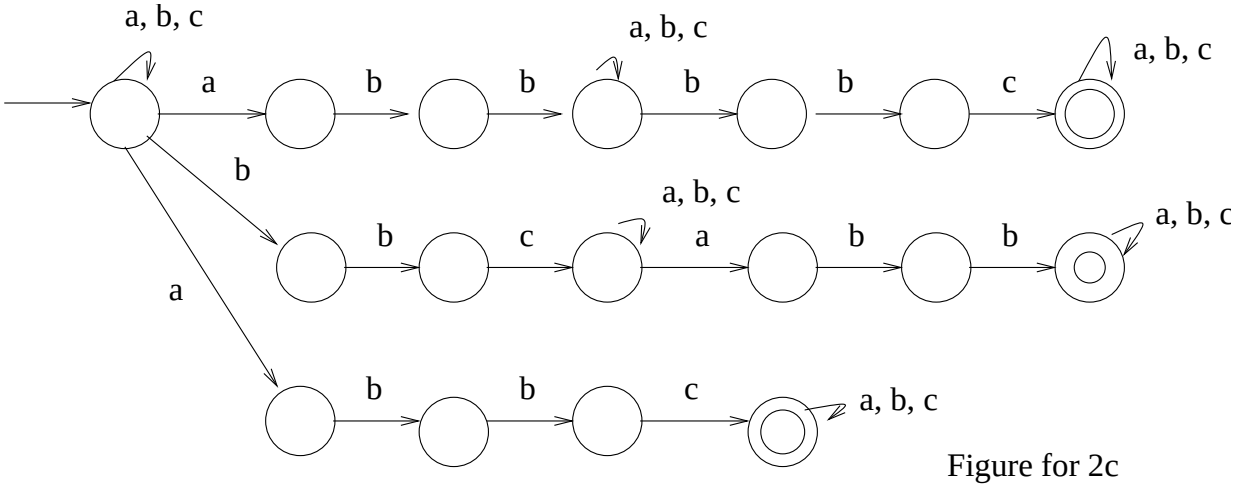


Figure for 2c

3. Describe concisely the language described by the regular expression.

   (a) $(a^*a)b \cup b$

   All strings of 0 or more a's followed by a single b.

   (b) $(a^*b^*)^*ab \cup (a^*b^*)^*ba(b \cup a)^*$

   All strings of a's and b's that contain either substring ba or end in ab.

4. Write a regular expression that represents the following languages.

   (a) $L = \{w \in \{a,b\}^* | \#_a(w) \leq 5\}$

   $b^*(a \cup \varepsilon)b^*(a \cup \varepsilon)b^*(a \cup \varepsilon)b^*(a \cup \varepsilon)b^*(a \cup \varepsilon)b^*$

   (b) $L = \{w \in \{a,b\}^* | \#_a(w) \equiv_3 0\}$

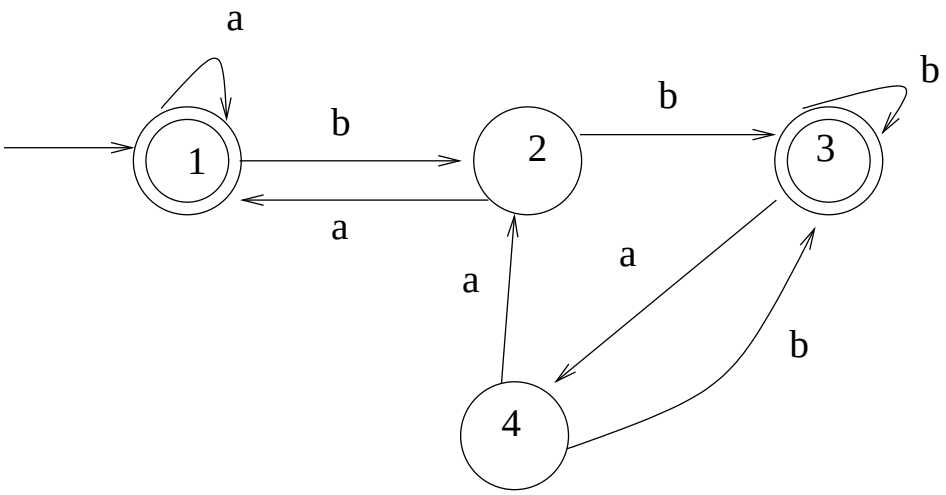
   $(b^*ab^*ab^*ab^*)^*b^*$

   (c) $L = \{w \in \{a,b\}^* | w$ contains exactly one occurrence of string aa$\}$

   $(ab \cup b)^*aa(ba \cup b)^*$

5. Use the algorithm presented in class to convert the regular expression $(0 \cup 1)^*000(0 \cup 1)^*$ to an NFA.

6. Convert the following finite automaton to a regular expression using the algorithm given in class.



7. 1.29 in Sipser
8. 1.30 in Sipser
9. 1.40a in Sipser
10. 1.46bd in Sipser
11. True or False:

    (a) The union of an infinite number of regular languages is regular.

    False. Recall that $L = \{0^{2^n} | n \geq 0\}$ is not regular. But
    $L = \{0^1\} \cup \{0^2\} \cup \{0^4\} \cup ...\}$, where each of the singleton sets $\{0^{2^i}\}$ (for fixed i) is regular.

    (b) The intersection of an infinite number of regular languages is regular.

    (c) Every subset of a regular language is regular.

    False. The set $L = \{a^n b^n | n \geq 0\}$ is a non-regular subset of regular language $(a \cup b)^*$.

12. 2.3 in Sipser
13. 2.4
14. 2.8
15. 2.6a
16. 2.30abc