CS341 Automata Theory

Summer 2007

CFLs Review Sheet

1. For each of the following languages, state whether the language is (I) regular, (II) context-free but not regular, or (III) not context-free. Prove your answer.

   (a) $\{0^n 0^{2n} 0^{3n} | n \geq 0\}$

   (b) $\{xwx^R | x, w \in \{0,1\}^+\}$

   (c) $\{xyz | x, y, z \in \{0,1\}^*, |x| = |z| > 0 \text{ and } \#_0(x) \geq \#_0(z)\}$

   (d) $\{1^{n^2} | n \geq 0\}$

   (e) $L(11(01 \cup 0)^*(01 \cup 0) \cup 10)$

   (f) $\{w\#w^R | w \in \{a,b\}^*\}$

   (g) $\{w | w \in \{a,b\}^*, w \neq w^R\}$

   (h) $\{a^i b^j c^k | i < j \rightarrow k < j\}$

2. Prove that the context-free languages are closed under union and concatenation. (Don't copy your class notes - this should be good practice for proving a closure property on the exam).

3. Suppose L_1, L_2 are context-free languages. Is it necessarily true that $L_1 - L_2$ is context-free? Prove your answer.

4. Convert the following grammar to Chomsky Normal Form. $S \rightarrow A|B$
   $A \rightarrow aA|aC$
   $B \rightarrow bB|bC$
   $C \rightarrow pqrW$
   $W \rightarrow TV$
   $T \rightarrow t|epsilon$

5. Show that the following CFG is ambiguous:
   $E \rightarrow E + E|E - E|ExE|(E)|a|b.$

6. For CFG G and string w, give an algorithm to decide if $w \in L(G)$.

7. Define a PDA that recognizes $\{0^n w 1^m | w \in \{0,1\}^*, \#_0(w) = \#_1(w) \text{ and } m \geq n\}$