Problem Set 7

CS 331

Due Wednesday, April 8

1. You are given a three dimensional object. On the horizontal plane it is an $n \times n$ square, and on the vertical axis each square (x, y) is a square pillar rising to height $h_{x,y} \geq 1$. Adjacent pillars, even ones sharing corners, are fused together.

   You submerse this object into a bucket of water, then carefully lift it out. Water will then drain off the sides, but it cannot drain through pillars. How many units of water will be captured in the object? Give an $O(n^2 \log n)$ algorithm.

   As an example, in the following grid 2 units will be captured, all in the center tile:

0	5	9
7	3	6
7	5	2

   Hint: You may use from class (on April 1) that a variant of Dijkstra's algorithm can solve the *minimax path* problem. The shortest path problem is to find paths minimizing total length $\sum_{e \in P} c(e)$; the minimax path problem is to find paths minimizing the *maximum* length $\max_{e \in P} c(e)$.

2. See the Jupyter notebook on the website. Warning: this one is more challenging than the previous ones.