CS 343H: Honors AI

Lecture 13:

Reinforcement Learning, part 2

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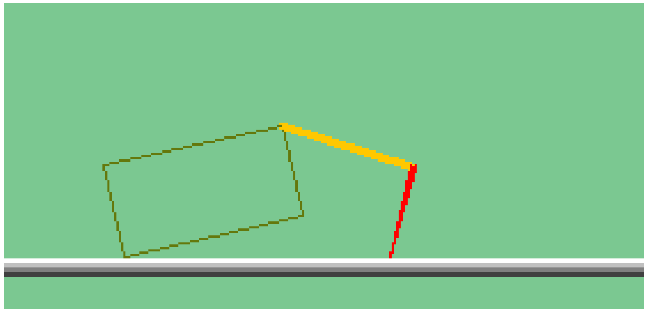
Slides courtesy of Dan Klein, UC Berkeley

Reinforcement Learning

- Still assume an MDP:
  - A set of states $s \in S$
  - A set of actions (per state) A
  - A model T(s,a,s')
  - A reward function R(s,a,s')
- Still looking for a policy $\pi(s)$
- New twist: don't know T or R



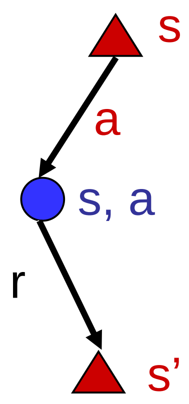
- **Big idea**: Compute all averages over T using sample outcomes

Recall: Model-Free Learning

- Model-free (temporal difference) learning
  - Experience world through episodes

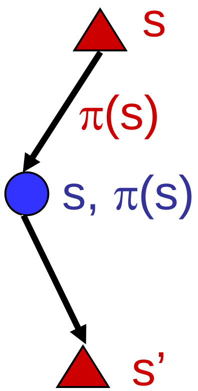
$$(s, a, r, s', a', r', s'', a'', r'', s'''' \ldots)$$

  - Update estimates each transition (s, a, r, s')
  - Over time, updates will mimic Bellman updates



Recall: Temporal-Difference Learning

- Big idea: learn from every experience!
  - Update V(s) each time we experience a transition (s,a,s',r)
  - Likely outcomes s' will contribute updates more often



- Temporal difference learning
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: "running average"

Sample of V(s): $$sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$$

Update to V(s): $$V^{\pi}(s) \leftarrow (1 - \alpha)V^{\pi}(s) + (\alpha) sample$$

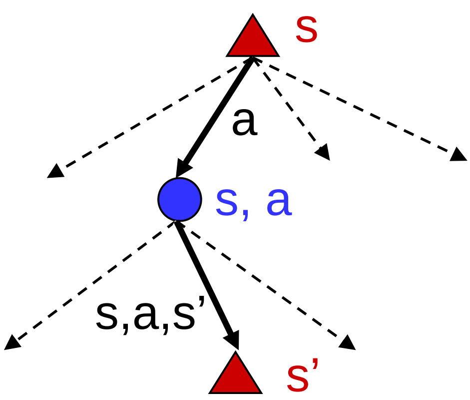
Same update: $$V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$$

Problems with TD Value Learning

- TD value leaning is model-free way to do policy evaluation, mimicking Bellman updates with running averages
- However, if we want to turn values into a (new) policy, we're sunk:

$$\pi(s) = \arg\max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$



- Idea: learn Q-values directly
- Makes action selection model-free too!

The Story So Far: MDPs and RL

Things we know how to do:

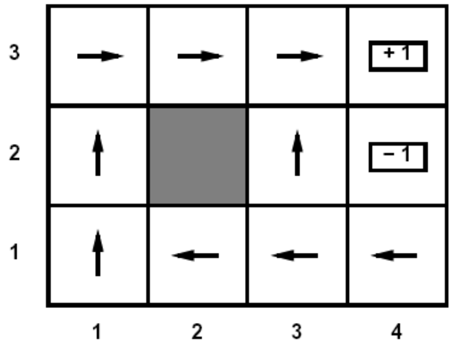
- If we know the MDP: offline
  - Compute V^*, Q^*, π^* exactly
  - Evaluate a fixed policy π
- If we don't know the MDP: online
  - We can estimate the MDP then solve
  - We can estimate V for a fixed policy π
  - We can estimate $Q^*(s,a)$ for the optimal policy while executing an exploration policy

Techniques:

- Model-based DPs
  - Value Iteration
  - Policy evaluation
- Model-based RL
- Model-free RL
  - Value learning
  - Q-learning

Active reinforcement learning

- Full reinforcement learning
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - You can choose any actions you like
  - Goal: learn the optimal policy / values



- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens…

Q-Value Iteration

- Value iteration: find successive depth-limited values
  - Start with $V_0(s) = 0$, which we know is right
  - Given Vi, calculate the depth i+1 values for all states:

$$V_{i+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_i(s') \right]$$

- But Q-values are more useful, so compute them instead
  - Start with $Q_0(s,a) = 0$, which we know is right
  - Given Qi, calculate the depth i+1 q-values for all q-states:

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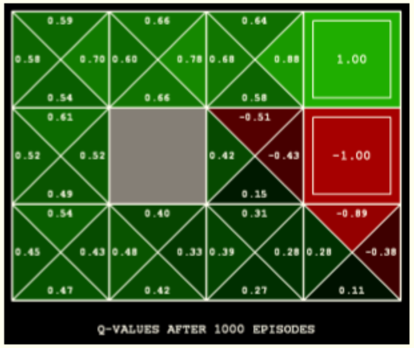
Q-Learning

- Q-Learning: sample-based Q-value iteration

$$Q_{i+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_i(s',a')\right]$$

- Learn Q(s,a) values as you go
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:



$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

  - Incorporate the new estimate into a running average:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[sample\right]$$

Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy, even if you're acting suboptimally!
- This is called off-policy learning.
- Caveats:
  - If you explore enough
  - If you make the learning rate small enough
  - … but not decrease it too quickly!
  - Basically in the limit it doesn't matter how you select actions (!)

The Story So Far: MDPs and RL

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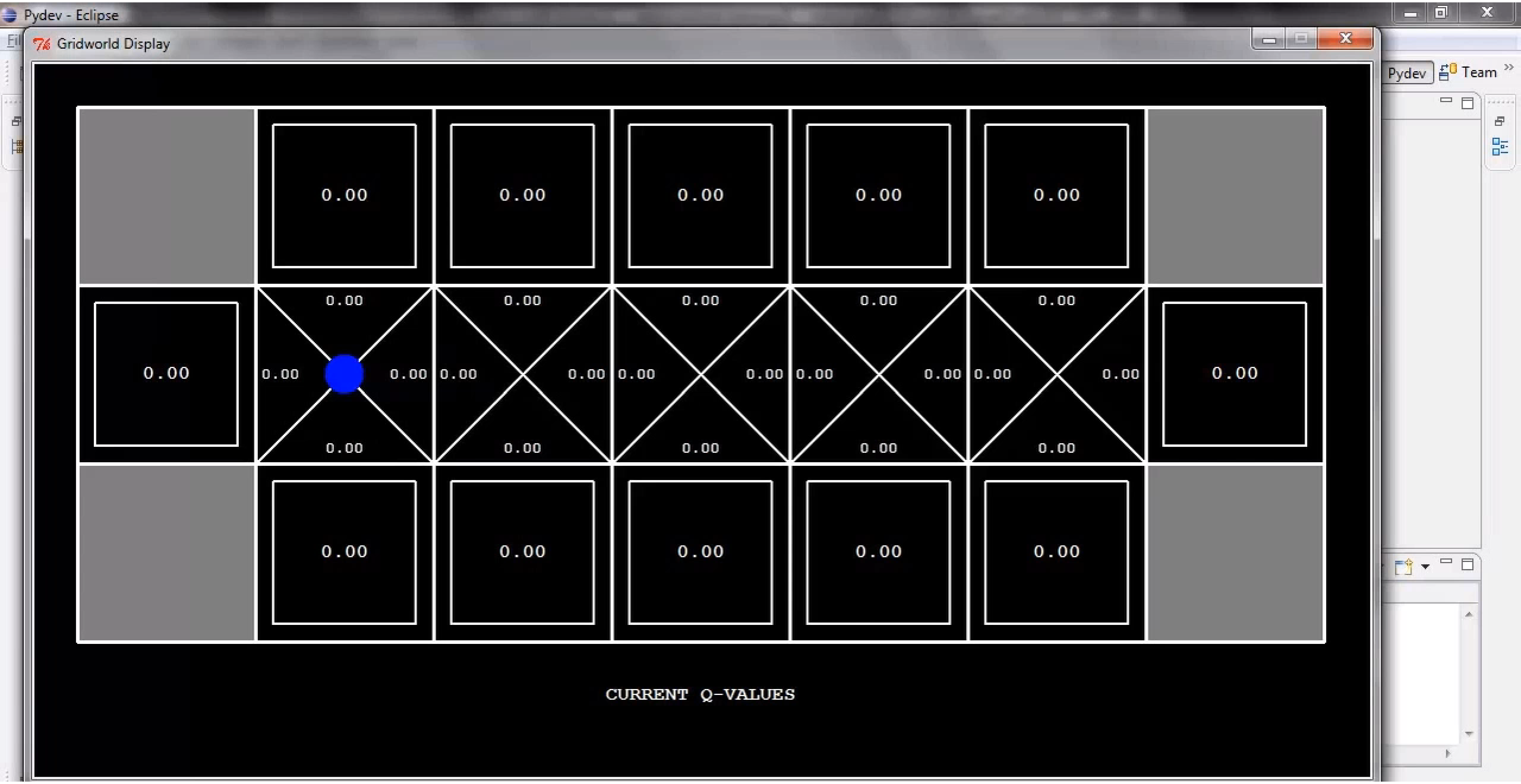
  - Value Iteration
  - Policy evaluation
- Model-based RL
- Model-free RL
  - Value learning
  - Q-learning

Exploring


Pydev - Eclipse
Gridworld Display
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CURRENT Q-VALUES
Pydev
Team

How to explore?

- Several schemes for forcing exploration
  - Simplest: random actions (ε greedy)
    - Every time step, flip a coin
    - With probability ε, act randomly
    - With probability 1-ε, act according to current policy

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- Problems with random actions?
  - You do eventually explore the space, but keep thrashing around once learning is done
  - One solution: lower ε over time
  - Another solution: exploration function

Exploration Functions

- When to explore?
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established, eventually stop exploring.
- Exploration function
  - Takes a value estimate and a visit count n, and returns an optimistic utility, e.g. $f(u,n) = u + k/n$

Regular Q-Update

$$Q_{i+1}(s,a) \leftarrow_\alpha R(s,a,s') + \gamma \max_{a'} Q_i(s',a')$$

Modified Q-Update

$$Q_{i+1}(s,a) \leftarrow_\alpha R(s,a,s') + \gamma \max_{a'} f(Q_i(s',a'), N(s',a'))$$

- Note: this propagates the 'bonus" back to states that lead to unknown states as well!

Regret

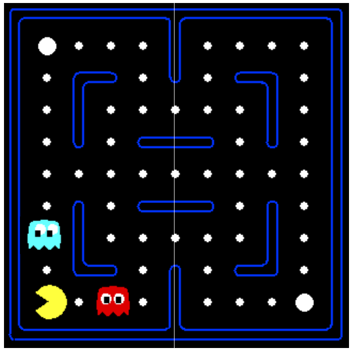
- Even if you learn the optimal policy, you still make mistakes along the way.
- Regret is a measure of your total mistake cost: difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards.
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal.
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret.

Generalizing across states

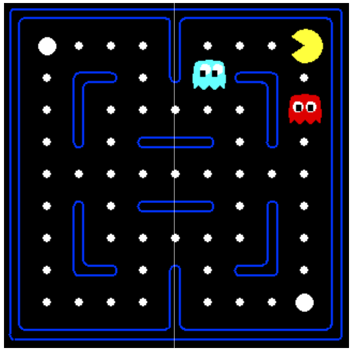
- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again

Example: Pacman

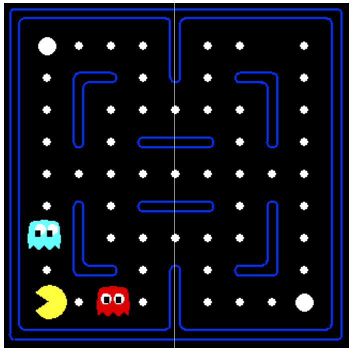
- Let's say we discover through experience that this state is bad:



- In naïve q learning, we know nothing about this state:

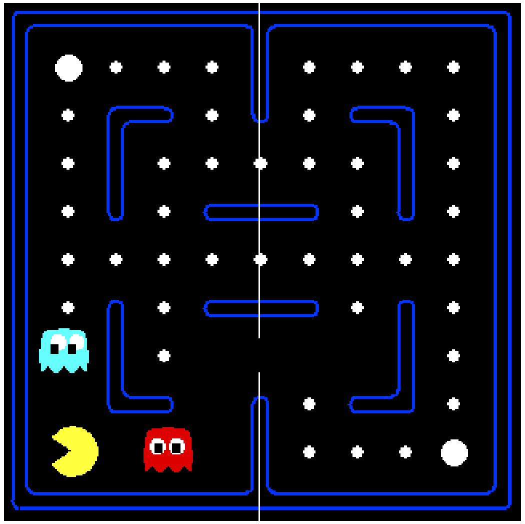


- Or even this one!



Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
    - Is it the exact state on this slide?
  - Can also describe a **q-state** (s, a) with features (e.g. action moves closer to food)



Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

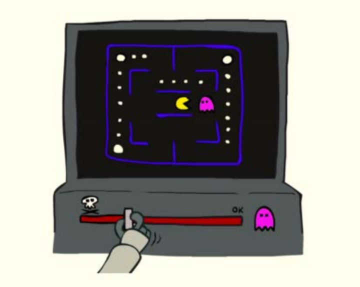
$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers

- Disadvantage: states may share features but actually be very different in value!

Approximate Q-learning



$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Q-learning with linear q-functions:

$$transition = (s, a, r, s')$$

$$\text{difference} = \left[r + \gamma \max_{a'} Q(s', a')\right] - Q(s,a)$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\, [\text{difference}]$$ Exact Q's

$$w_i \leftarrow w_i + \alpha\, [\text{difference}]\, f_i(s,a)$$ Approximate Q's

- Intuitive interpretation:
  - Adjust weights of active features
  - E.g. if something unexpectedly bad happens, we start to prefer <u>less</u> all states with that state's features

Example: Pacman with approx. Q-learning

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$

$$f_{DOT}(s, \text{NORTH}) = 0.5$$

$$f_{GST}(s, \text{NORTH}) = 1.0$$

$$Q(s,a) = +1$$

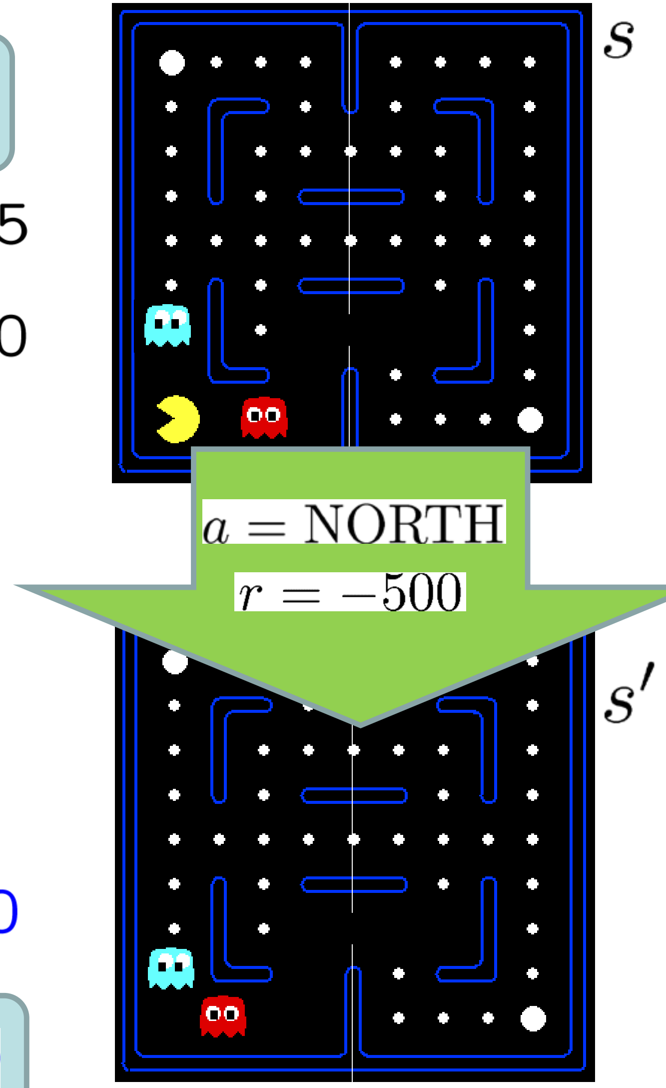
$$R(s,a,s') = -500$$

$$\left[r + \gamma \max_{a'} Q(s',a')\right] - Q(s,a) \longrightarrow difference = -501$$

$$w_{DOT} \leftarrow 4.0 + \alpha \left[-501\right] 0.5$$

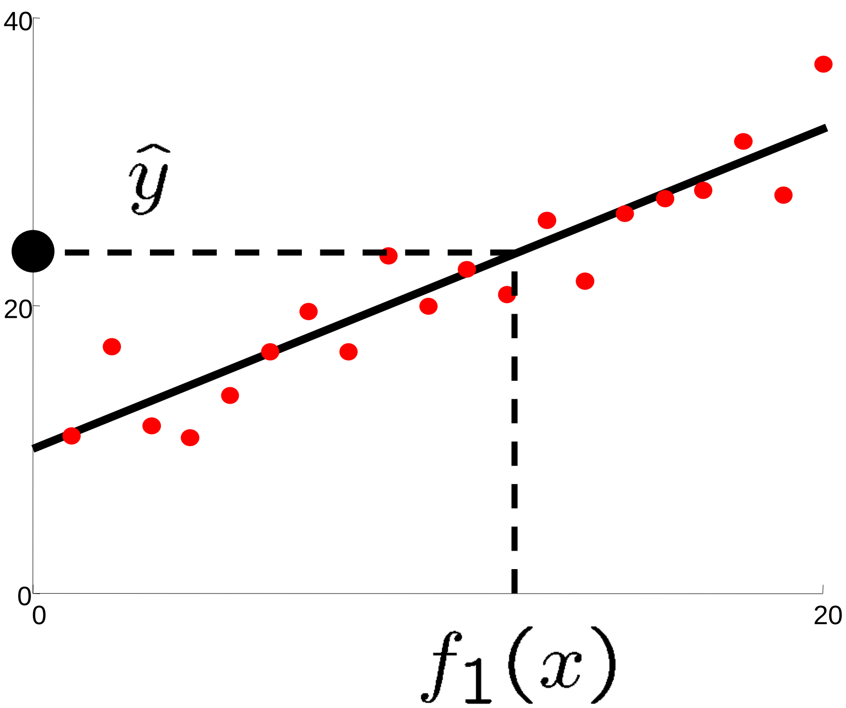
$$w_{GST} \leftarrow -1.0 + \alpha \left[-501\right] 1.0$$

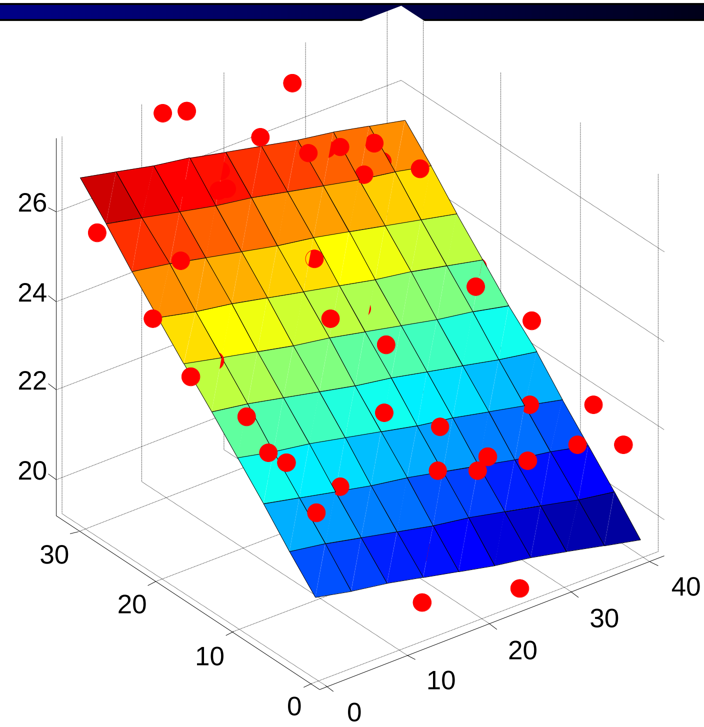
$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$



Q(s', -) = 0

Linear approximation: Regression





Prediction

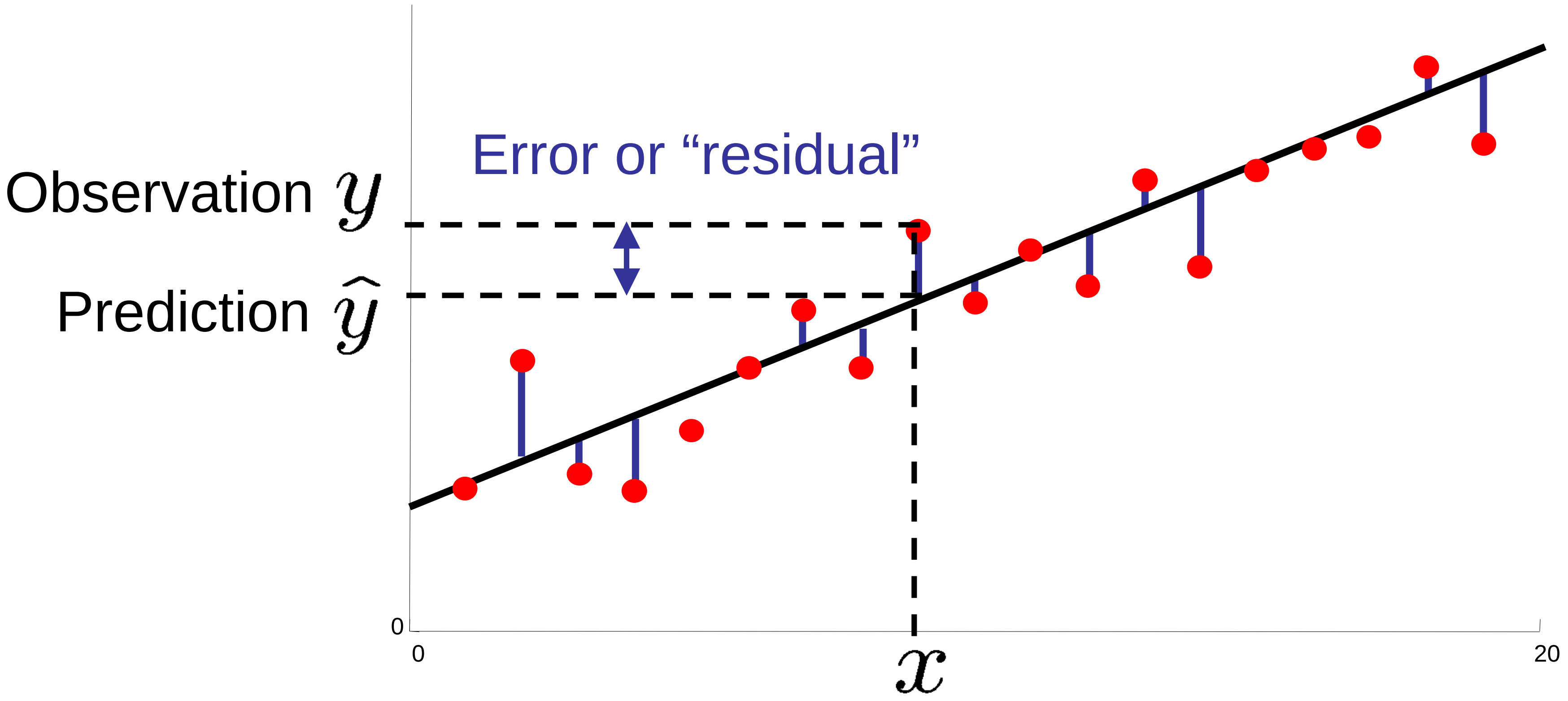
$$\widehat{y} = w_0 + w_1 f_1(x)$$

Prediction

$$\widehat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least squares

$$\text{total error} = \sum_i (y_i - \widehat{y}_i)^2 = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$$



Minimizing Error

Imagine we had only one point x with features f(x), target value y, and weights w:

$$\text{error}(w) = \frac{1}{2}\left(y - \sum_k w_k f_k(x)\right)^2$$

$$\frac{\partial\ \text{error}(w)}{\partial w_m} = -\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

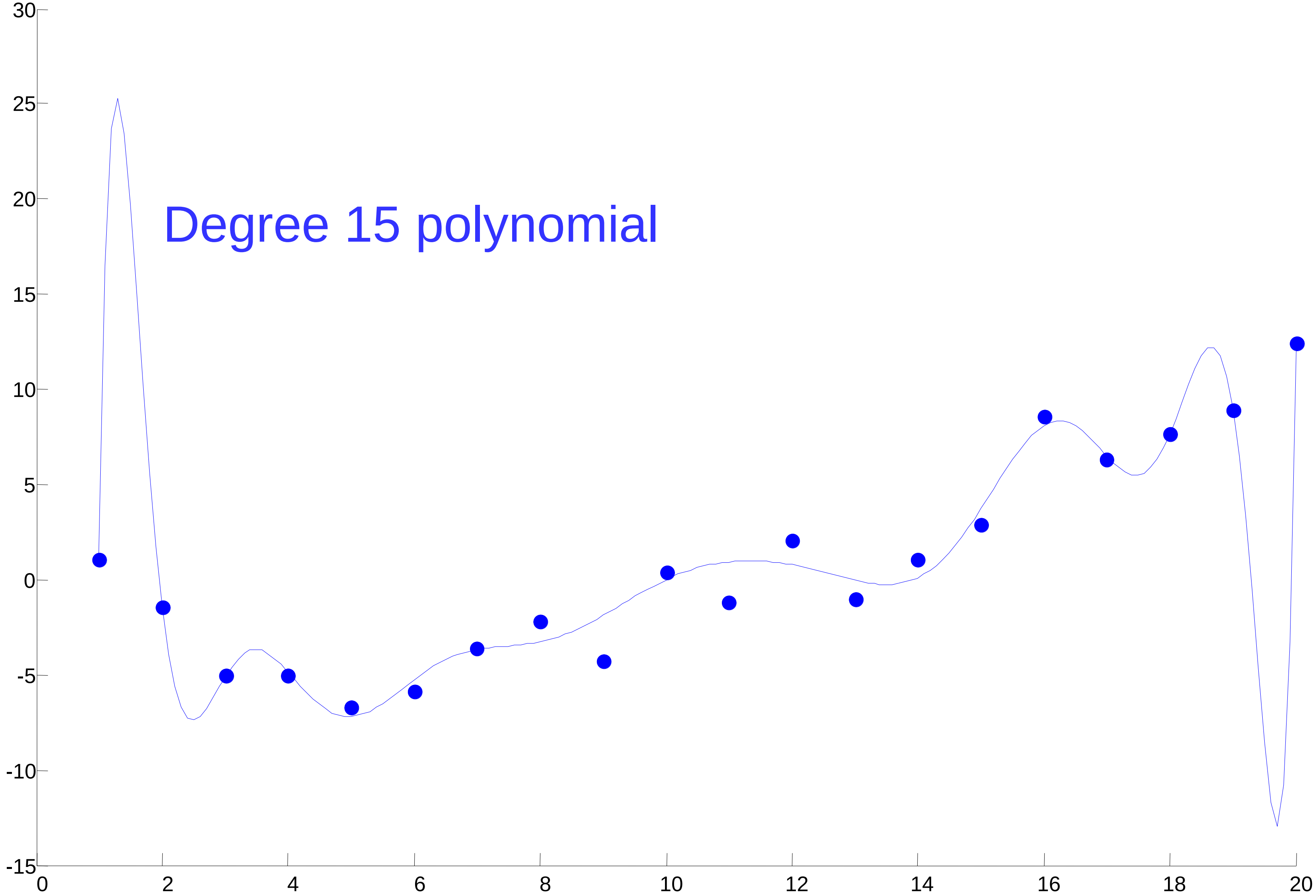
$$w_m \leftarrow w_m + \alpha\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

Approximate q update explained:

“target” “prediction”

$$w_m \leftarrow w_m + \alpha\left[r + \gamma \max_a Q(s', a') - Q(s, a)\right] f_m(s, a)$$

Overfitting: why limiting capacity can help



Policy Search

- **Problem**: Often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate V / Q best
  - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
  - Q-learning's priority: get Q-values close (modeling)
  - Action selection priority: get ordering of Q-values right (prediction)
  - We'll see this distinction between modeling and prediction again later in the course

- **Solution**: learn the <u>policy</u> that maximizes rewards rather than the <u>value</u> that predicts rewards

- **Policy search**: start with an ok solution (e.g., Q learning), then fine-tune by hill climbing on feature weights.

Policy Search

- Simplest policy search:
  - Start with an initial linear value function or q-function
  - Nudge each feature weight up and down and see if your policy is better than before

- Problems:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical

- Better methods exploit lookahead structure, sample wisely, change multiple parameters…

Take a Deep Breath…

- We’re done with search and planning!

- Next, we’ll look at how to reason with probabilities
  - Diagnosis
  - Tracking objects
  - Speech recognition
  - Robot mapping
  - … lots more!

- Last part of course: machine learning