Perceptrons

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(Slides taken from Dan Klein)

Classification: Feature Vectors

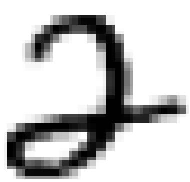
x | $f(x)$ | y

Hello,

Do you want free printr cartriges? Why pay more when you can get them ABSOLUTELY FREE! Just

 # free : 2
YOUR_NAME : 0
MISSPELLED : 2
FROM_FRIEND : 0
...

SPAM
or
+



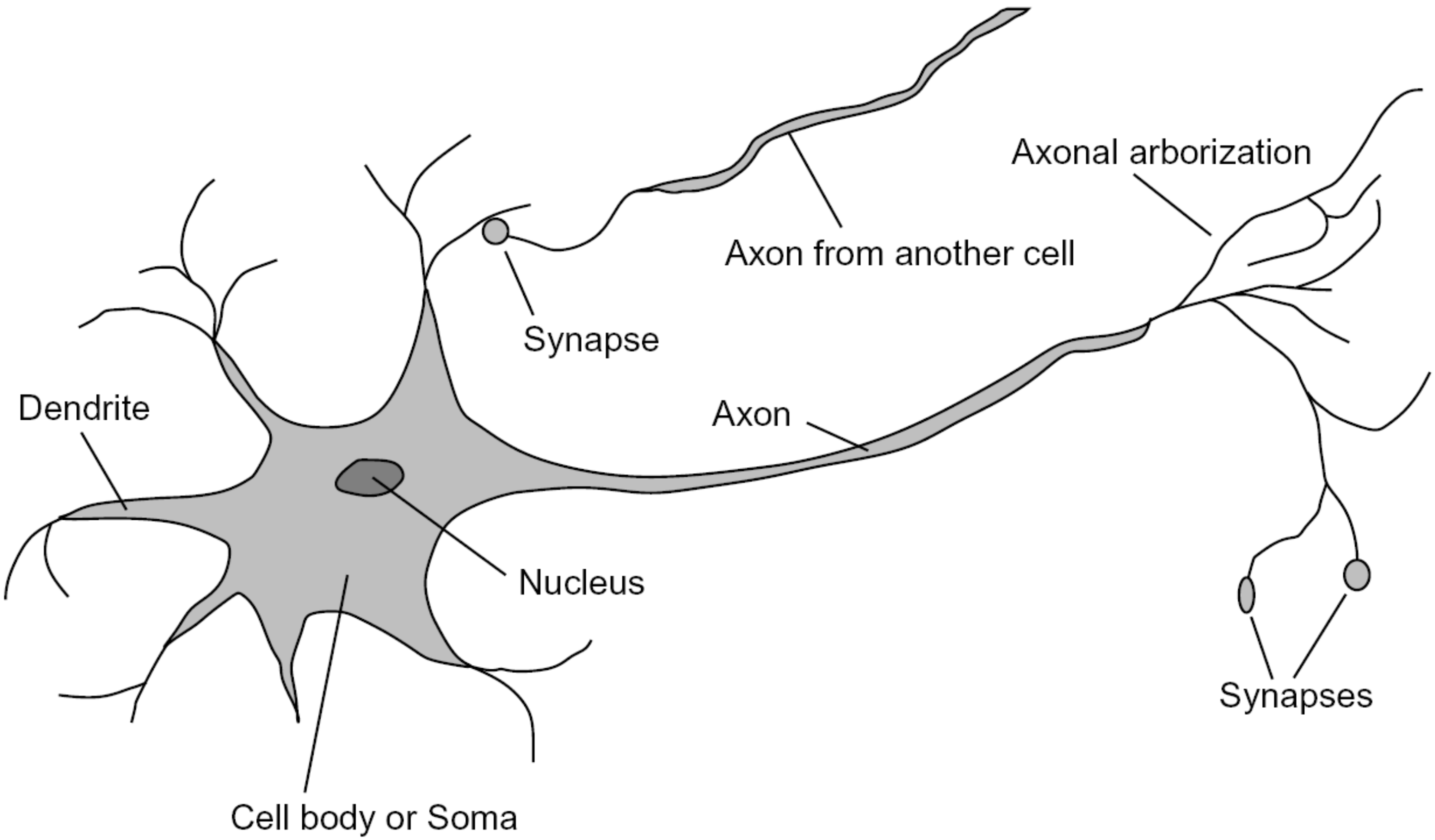
PIXEL-7,12 : 1
PIXEL-7,13 : 0
...
NUM_LOOPS : 1
...

“2”

This slide deck courtesy of Dan Klein at UC Berkeley

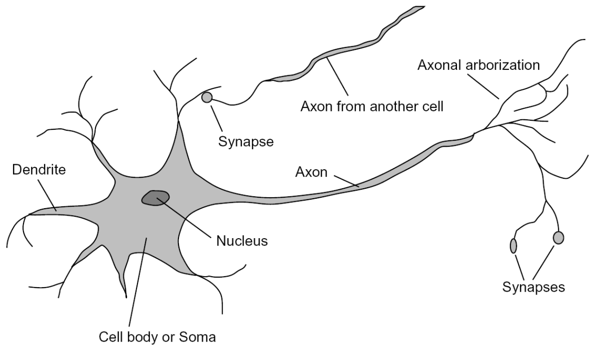
Some (Simplified) Biology

- Very loose inspiration: human neurons



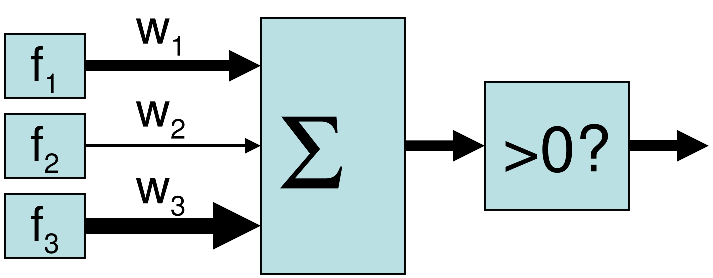
Linear Classifiers

- Inputs are feature values
- Each feature has a weight
- Sum is the activation



$$\text{activation}_w(x) = \sum_i w_i \cdot f_i(x) = w \cdot f(x)$$

- If the activation is:
  - Positive, output +1
  - Negative, output -1



Example: Spam

- Imagine 4 features (spam is "positive" class):
  - free (number of occurrences of "free")
  - money (occurrences of "money")
  - BIAS (intercept, always has value 1)

$$w \cdot f(x)$$

↑

$$\sum_i w_i \cdot f_i(x)$$

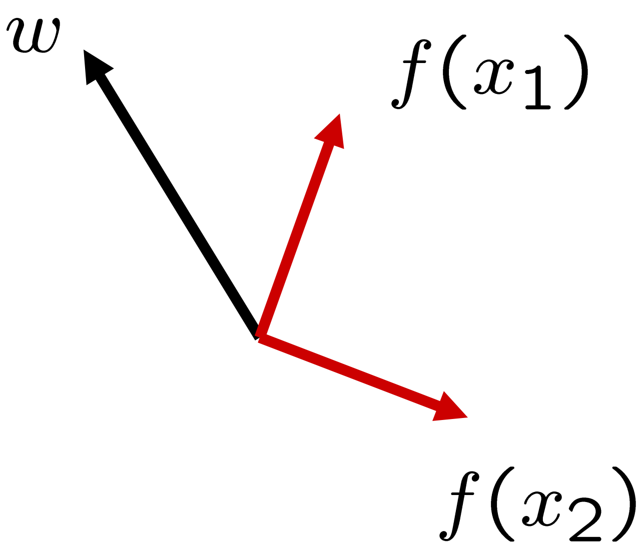
x	$f(x)$	w
"free money"	BIAS : 1 free : 1 money : 1 ...	BIAS : -3 free : 4 money : 2 ...

$$(1)(-3) \; +$$
$$(1)(4) \; +$$
$$(1)(2) \; +$$
$$\ldots$$
$$= 3$$

Classification: Weights

- Binary case: compare features to a weight vector
- Learning: figure out the weight vector from examples

```
# free        :  4
YOUR_NAME     : -1
MISSPELLED    :  1
FROM_FRIEND   : -3
...
```



$f(x_1)$

```
# free        : 2
YOUR_NAME     : 0
MISSPELLED    : 2
FROM_FRIEND   : 0
...
```

$f(x_2)$

```
# free        : 0
YOUR_NAME     : 1
MISSPELLED    : 1
FROM_FRIEND   : 1
...
```

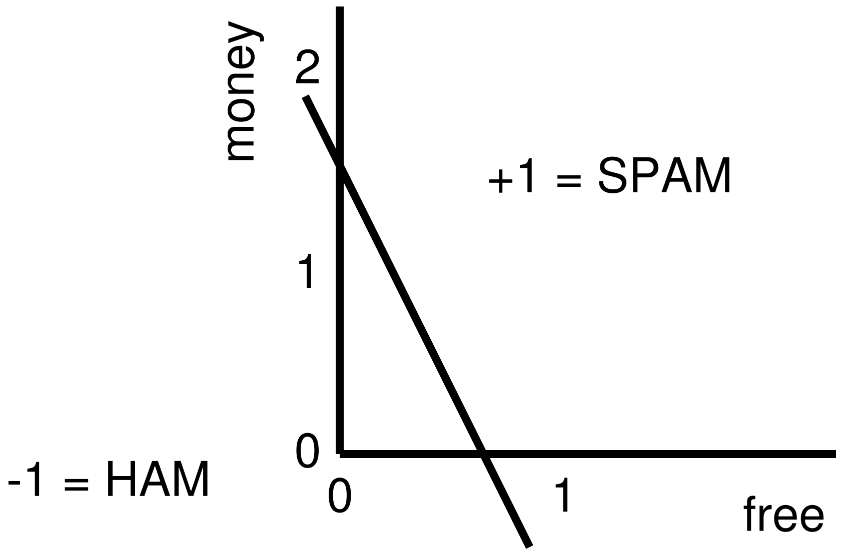
Dot product $w \cdot f$ *positive means the positive class*

Binary Decision Rule

- In the space of feature vectors
  - Examples are points
  - Any weight vector is a hyperplane
  - One side corresponds to Y=+1
  - Other corresponds to Y=-1

w

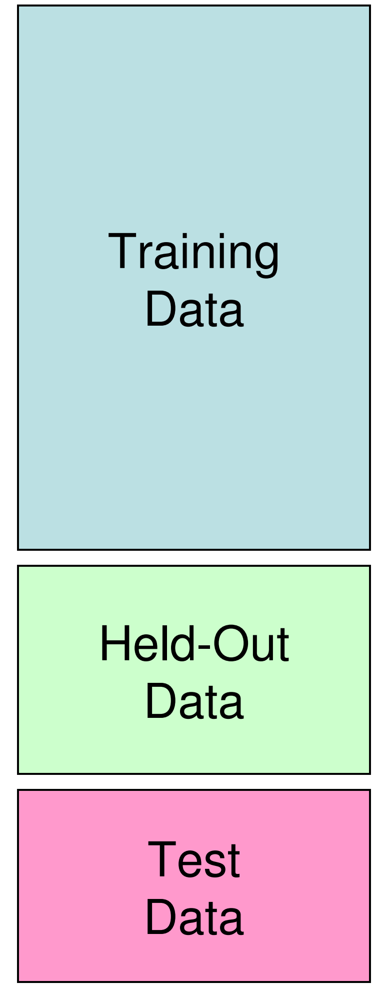
BIAS	:	-3
free	:	4
money	:	2
...		



$f \cdot w = 0$

Mistake-Driven Classification

- For Naïve Bayes:
  - Parameters from data statistics
  - Parameters: causal interpretation
  - Training: one pass through the data
- For the perceptron:
  - Parameters from reactions to mistakes
  - Prameters: discriminative interpretation
  - Training: go through the data until held-out accuracy maxes out

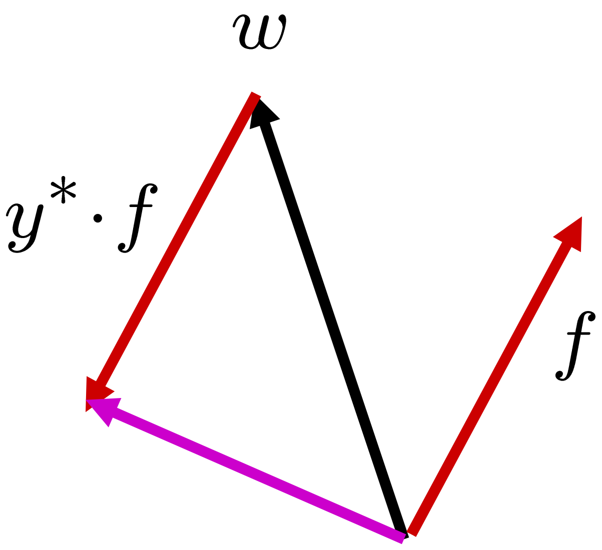


Learning: Binary Perceptron

- Start with weights = 0
- For each training instance:
  - Classify with current weights

$$y = \begin{cases} +1 & \text{if } \; w \cdot f(x) \geq 0 \\ -1 & \text{if } \; w \cdot f(x) < 0 \end{cases}$$

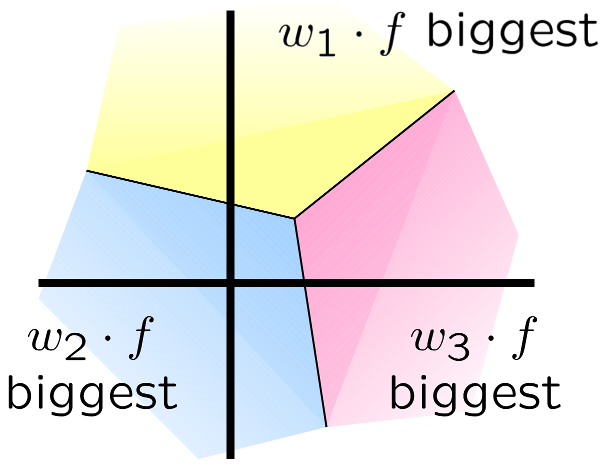
  - If correct (i.e., y=y*), no change!
  - If wrong: adjust the weight vector by adding or subtracting the feature vector. Subtract if y* is -1.



$$w = w + y^* \cdot f$$

Multiclass Decision Rule

- If we have more than two classes:
  - Have a weight vector for each class: w_y
  - Calculate an activation for each class



$$\text{activation}_w(x, y) = w_y \cdot f(x)$$

  - Highest activation wins

$$y = \arg\max_y \left(\text{activation}_w(x, y)\right)$$

Multiclass Decision Rule

- If we have multiple classes:
  - A weight vector for each class:

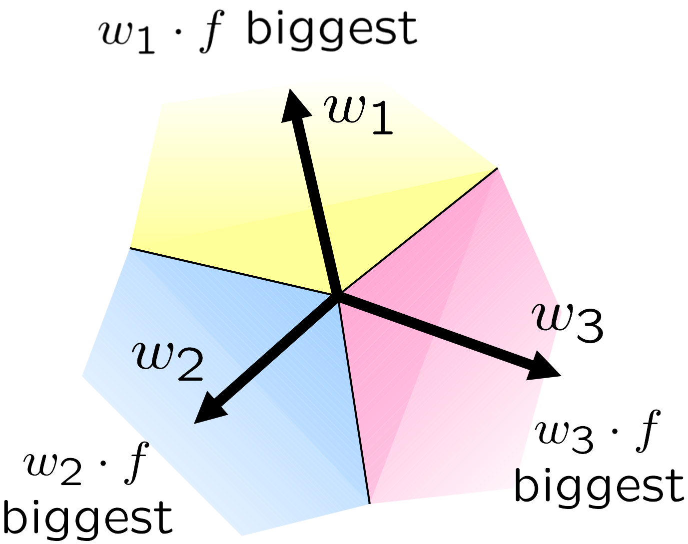
    $$w_y$$

  - Score (activation) of a class y:

    $$w_y \cdot f(x)$$

  - Prediction highest score wins

    $$y = \arg\max_y \; w_y \cdot f(x)$$



Binary = multiclass where the negative class has weight zero

Example

"win the vote" →

```
BIAS  :  1
win   :  1
game  :  0
vote  :  1
the   :  1
...
```

w_{SPORTS}

```
BIAS  : -2
win   :  4
game  :  4
vote  :  0
the   :  0
...
```

$w_{POLITICS}$

```
BIAS  :  1
win   :  2
game  :  0
vote  :  4
the   :  0
...
```

w_{TECH}

```
BIAS  :  2
win   :  0
game  :  2
vote  :  0
the   :  0
...
```

Learning: Multiclass Perceptron

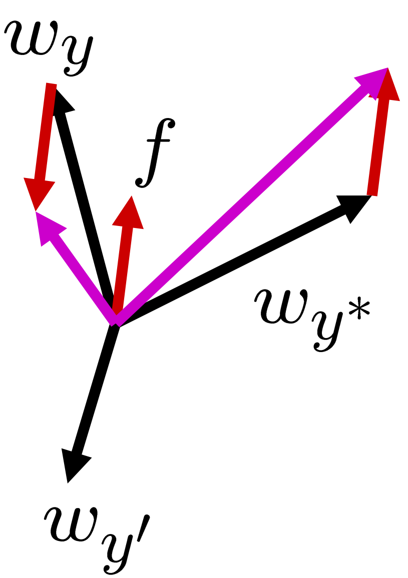
- Start with all weights = 0
- Pick up training examples one by one
- Predict with current weights

$$y = \arg\max_y \; w_y \cdot f(x)$$

- If correct, no change!
- If wrong: lower score of wrong answer, raise score of right answer

$$w_y = w_y - f(x)$$

$$w_{y^*} = w_{y^*} + f(x)$$



Example: Multiclass Perceptron

"win the vote"

"win the election"

"win the game"

w_{SPORTS}

BIAS	:	1
win	:	0
game	:	0
vote	:	0
the	:	0
...		

$w_{POLITICS}$

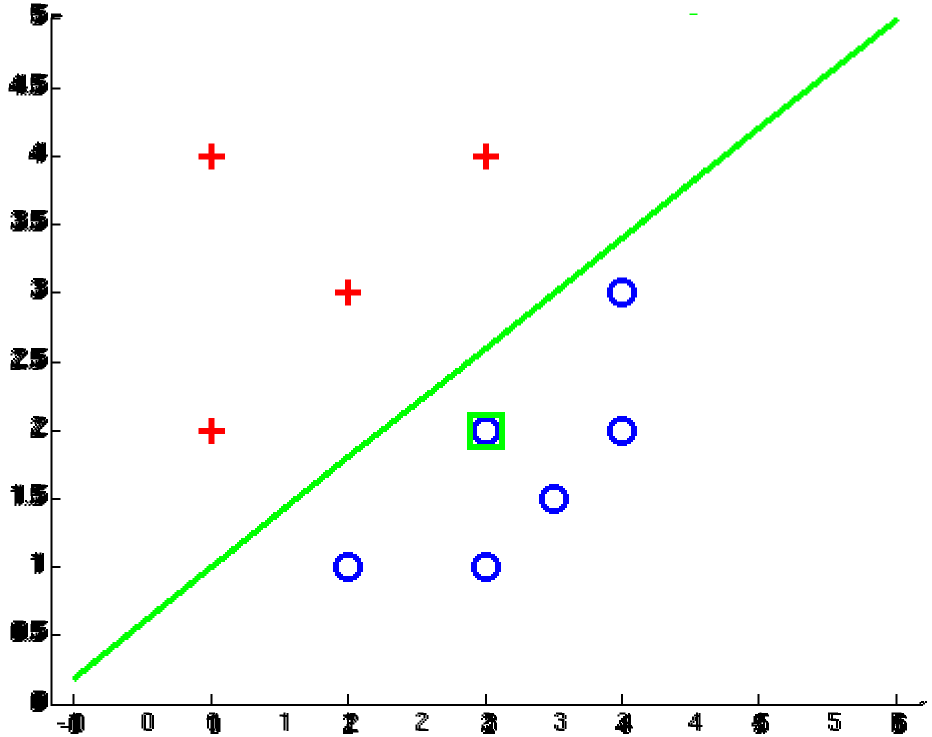
BIAS	:	0
win	:	0
game	:	0
vote	:	0
the	:	0
...		

w_{TECH}

BIAS	:	0
win	:	0
game	:	0
vote	:	0
the	:	0
...		

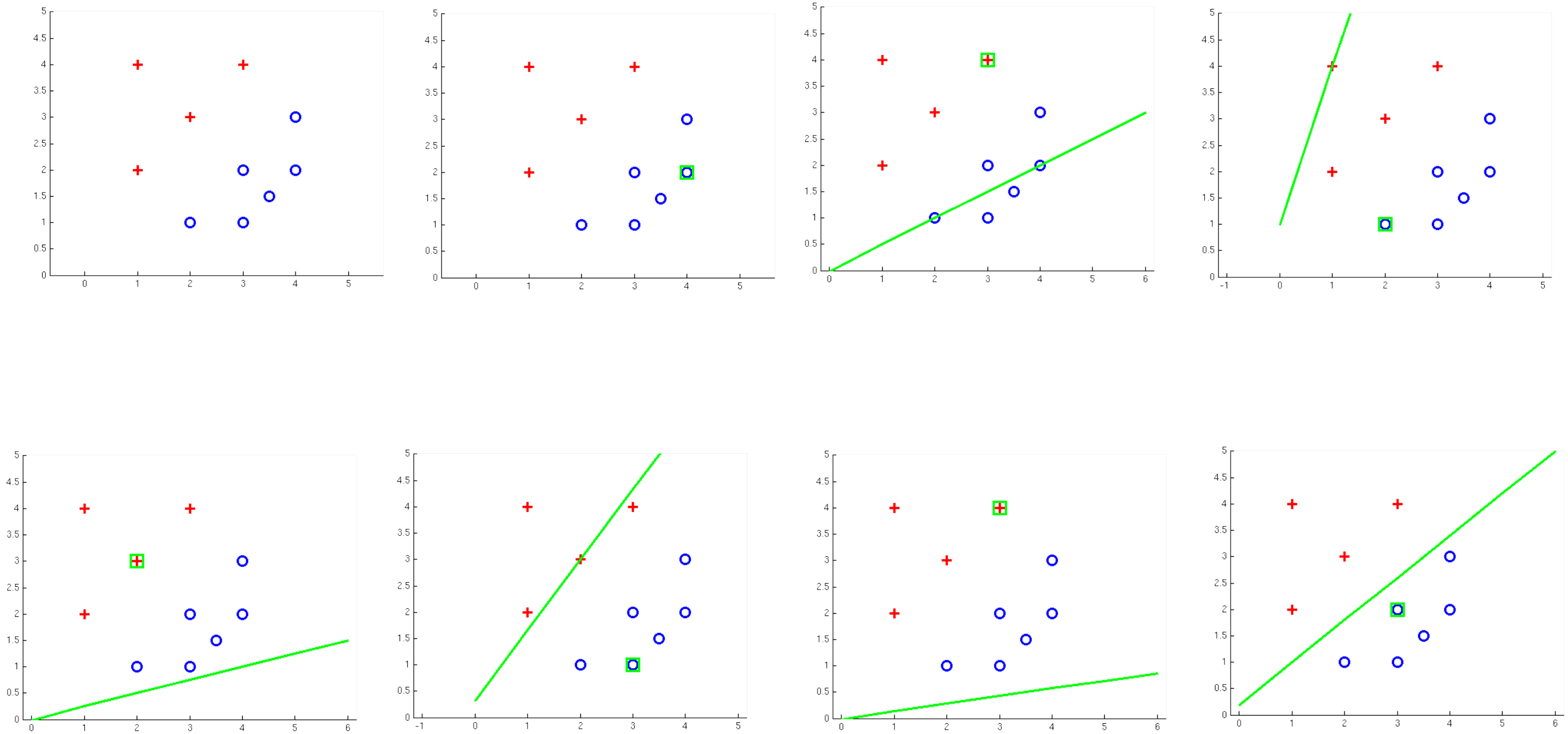
Examples: Perceptron

- Separable Case



Examples: Perceptron

- Separable Case

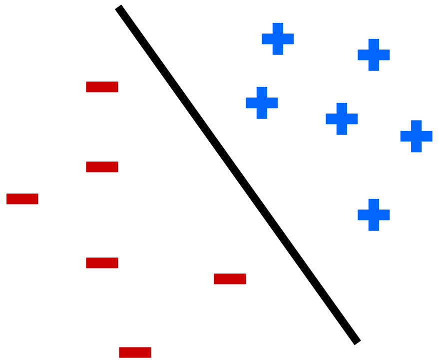


Properties of Perceptrons

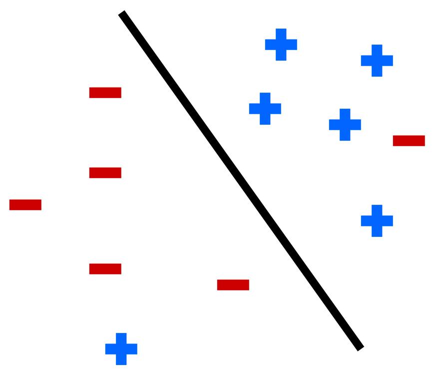
- Separability: some parameters get the training set perfectly correct
- Convergence: if the training is separable, perceptron will eventually converge (binary case)
- Mistake Bound: the maximum number of mistakes (binary case) related to the *margin* or degree of separability

$$\text{mistakes} < \frac{k}{\delta^2}$$

Separable

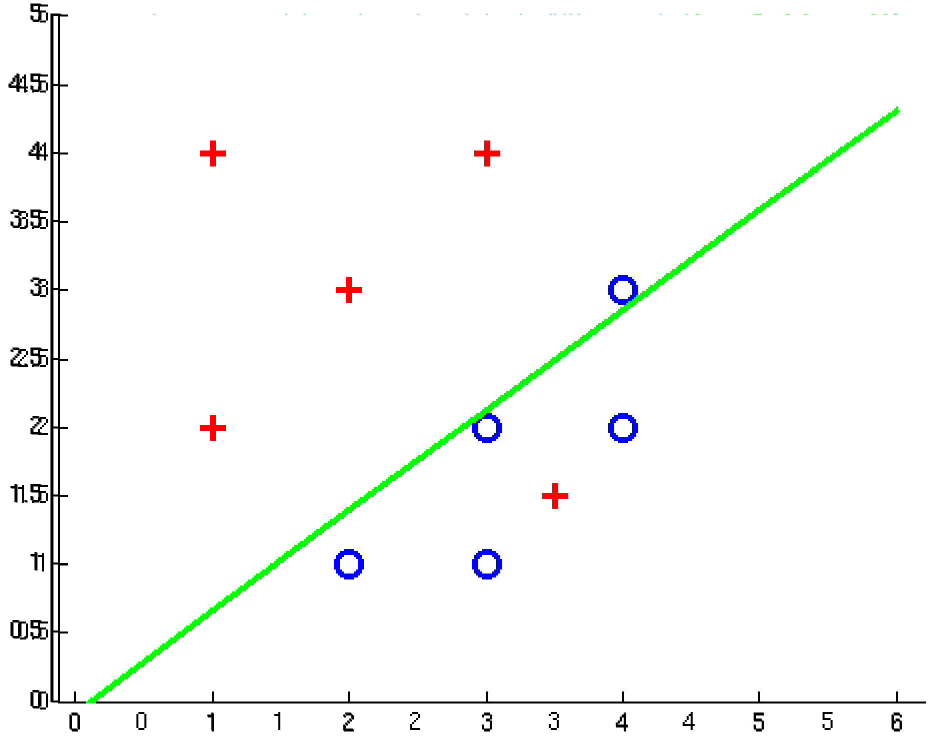


Non-Separable



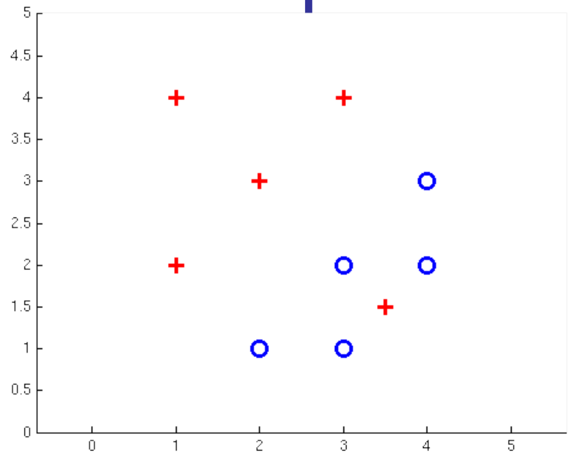
Examples: Perceptron

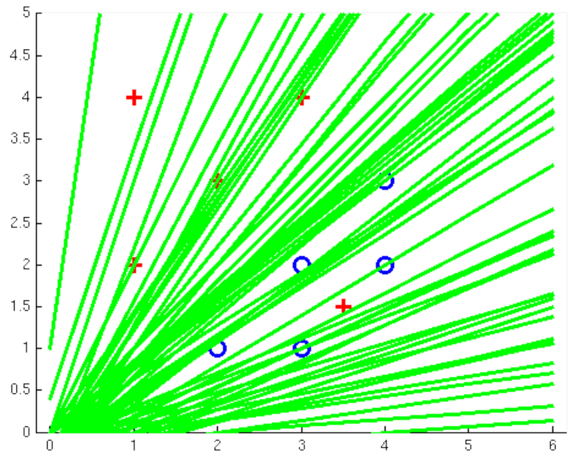
- Non-Separable Case

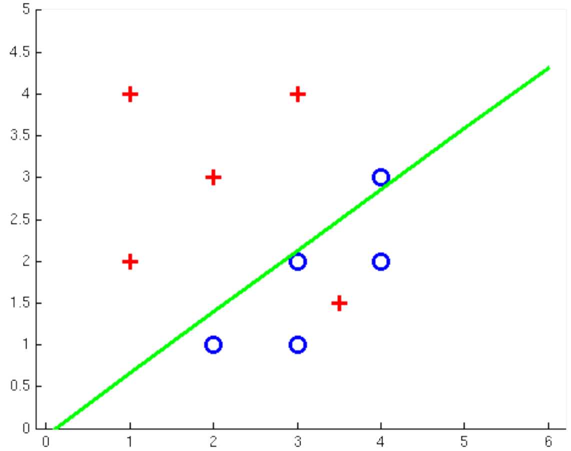


Examples: Perceptron

- Non-Separable Case

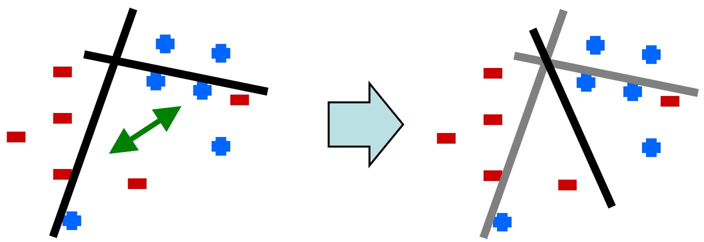




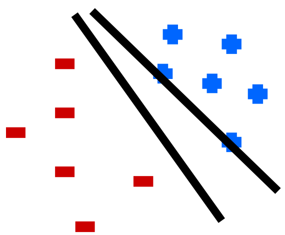


Problems with the Perceptron

- Noise: if the data isn't separable, weights might thrash
  - Averaging weight vectors over time can help (averaged perceptron)



- Mediocre generalization: finds a "barely" separating solution



- Overtraining: test / held-out accuracy usually rises, then falls
  - Overtraining is a kind of overfitting