Copyright

by

Mikhail Yuryevich Bilenko

2006

The Dissertation Committee for Mikhail Yuryevich Bilenko certifies that this is the approved version of the following dissertation:

# Learnable Similarity Functions and Their Application to Record Linkage and Clustering

Committee:

Raymond J. Mooney, Supervisor

William W. Cohen

Inderjit S. Dhillon

Joydeep Ghosh

Peter H. Stone

# Learnable Similarity Functions and Their Application to Record Linkage and Clustering

by

Mikhail Yuryevich Bilenko, B.S.; M.S.

### Dissertation

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

## **Doctor of Philosophy**

# The University of Texas at Austin

August 2006

To my dear grandparents

# Acknowledgments

I am indebted to many people for all the help and support I received over the past six years. Most notably, no other person influenced me as much as my advisor, Ray Mooney. Ray has been a constant source of motivation, ideas, and guidance. His encouragement to explore new directions while maintaining focus has kept me productive and excited about research. Ray's passion about science, combination of intellectual depth and breadth, and great personality have all made working with him a profoundly enriching and fun experience.

Members of my committee, William Cohen, Inderjit Dhillon, Joydeep Ghosh and Peter Stone, have provided me with many useful ideas and comments. William's work has been a great inspiration for applying machine learning methods to record linkage, and I am very grateful to him for providing detailed feedback and taking the time to visit Austin. Inderjit and Joydeep gave me a terrific introduction to data mining in my first year in graduate school, and have had a significant influence on me through their work on clustering. I deeply appreciate Peter's helpful insights, encouragement and advice on matters both related and unrelated to this research.

I have been lucky to work with several wonderful collaborators. Throughout my graduate school years, Sugato Basu has been the perfect partner on countless projects, a great sounding board, and a kind friend. Talking to Arindam Banerjee has always been both eye-opening and motivating. During my internship at Google, working with Mehran Sahami has been enlightening and exciting, and I have appreciated Mehran's kindness and wisdom on many occasions. Exploring several new research directions in the past year with

Beena Kamath, Peng Bi, Tuyen Huynh, Maytal Saar-Tsechansky and Duy Vu has been a stimulating finish for my graduate career.

My academic siblings in the Machine Learning Group are a wonderful clever bunch who have been a joy to be around. I am most grateful to Prem Melville for his trusty friendship, razor-sharp wit, and reliable company in all sorts of places at all sorts of hours; *doing science* (and not-quite-science) side-by-side with Prem has been a highlight of the past years. Lily Mihalkova and Razvan Bunescu have been essential in providing much needed moral support in the office when it was most needed in the final stretch. Sugato, Prem, Lily, Razvan, Un Yong Nahm, John Wong, Ruifang Ge, Rohit Kate, and Stewart Yang have been always ready to help with papers, practice talks and anything else in work and life, and I hope to see them often for years to come.

A number of UTCS staff members have provided me their support during the past six years. The administrative expertise of Stacy Miller, Gloria Ramirez and Katherine Utz has been much appreciated on many occasions. My life has been made much easier by the friendly department computing staff who have provided a well-oiled and smoothly running environment for all the experiments in this thesis.

I am grateful to many fellow UTCS grad students for their intellectually stimulating camaraderie. Joseph Modayil has been a great friend, a dream roommate, and baker extraordinaire whose editing skills have been invaluable a number of times, including the present moment. Yuliya Lierler's warm friendship has brightened many days, and seeing her in Austin after her German sojourn is a perfect graduation gift. The company of Aniket Murarka, Amol Nayate, Nalini Belaramani, Patrick Beeson, Nick Jong, Alison Norman and Serita Nelesen has been a precious pleasure on too many occasions to count, and I hope I will enjoy it in the future as often as possible.

My friends outside computer science have made my years in Austin a fantastic experience. From climbing trips to live music shows to random coffeeshop jaunts, they have given my life much-needed balance and spice. My road to this point would not be possible without the people I love. My sister Natalia, parents Olga and Yury, and grandparents Tamara Alexandrovna, Nina Alexandrovna, Alexander Mikhailovich and David Isakovich have always given me their unconditional love and support. My family has inspired me, encouraged me, and helped me in every possible way, and there are no words to express my gratitude. Sandra, Dick, Gretchen and Grant Fruhwirth selflessly opened their home to me when I first came to the US, have been by my side during my college years, and will always be family to me. Finally, I thank Anna Zaster for making my life complete. I would not be happy without you.

The research in this thesis was supported by the University of Texas MCD Fellowship, the National Science Foundation under grants IIS-0117308 and EIA-0303609, and a Google Research Grant.

#### MIKHAIL YURYEVICH BILENKO

The University of Texas at Austin August 2006

# Learnable Similarity Functions and Their Application to Record Linkage and Clustering

Publication No.

Mikhail Yuryevich Bilenko, Ph.D. The University of Texas at Austin, 2006

Supervisor: Raymond J. Mooney

Many machine learning and data mining tasks depend on functions that estimate similarity between instances. Similarity computations are particularly important in clustering and information integration applications, where pairwise distances play a central role in many algorithms. Typically, algorithms for these tasks rely on pre-defined similarity measures, such as edit distance or cosine similarity for strings, or Euclidean distance for vector-space data. However, standard distance functions are frequently suboptimal as they do not capture the appropriate notion of similarity for a particular domain, dataset, or application.

In this thesis, we present several approaches for addressing this problem by employing *learnable* similarity functions. Given supervision in the form of similar or dissimilar pairs of instances, learnable similarity functions can be trained to provide accurate estimates for the domain and task at hand. We study the problem of adapting similarity functions in the context of several tasks: record linkage, clustering, and blocking. For each of these tasks, we present learnable similarity functions and training algorithms that lead to improved performance.

In record linkage, also known as duplicate detection and entity matching, the goal is to identify database records referring to the same underlying entity. This requires estimating similarity between corresponding field values of records, as well as overall similarity between records. For computing field-level similarity between strings, we describe two learnable variants of edit distance that lead to improvements in linkage accuracy. For learning record-level similarity functions, we employ Support Vector Machines to combine similarities of individual record fields in proportion to their relative importance, yielding a high-accuracy linkage system. We also investigate strategies for efficient collection of training data which can be scarce due to the pairwise nature of the record linkage task.

In clustering, similarity functions are essential as they determine the grouping of instances that is the goal of clustering. We describe a framework for integrating learnable similarity functions within a probabilistic model for semi-supervised clustering based on Hidden Markov Random Fields (HMRFs). The framework accommodates learning various distance measures, including those based on Bregman divergences (e.g., parameterized Mahalanobis distance and parameterized KL-divergence), as well as directional measures (e.g., cosine similarity). Thus, it is applicable to a wide range of domains and data representations. Similarity functions are learned within the HMRF-KMEANS algorithm derived from the framework, leading to significant improvements in clustering accuracy.

The third application we consider, blocking, is critical in making record linkage and clustering algorithms scalable to large datasets, as it facilitates efficient selection of approximately similar instance pairs without explicitly considering all possible pairs. Previously proposed blocking methods require manually constructing a similarity function or a set of similarity predicates, followed by hand-tuning of parameters. We propose learning blocking functions automatically from linkage and semi-supervised clustering supervision, which allows automatic construction of blocking methods that are efficient and accurate. This approach yields computationally cheap learnable similarity functions that can be used for scaling up in a variety of tasks that rely on pairwise distance computations, including record linkage and clustering.

# Contents

Acknow	ledgments	V
Abstrac	t	viii
List of I	ligures	xiv
Chapter	1 Introduction	1
1.1	Motivation	1
1.2	Thesis Contributions	4
1.3	Thesis Outline	5
Chapter	2 Background	7
2.1	Similarity functions	8
	2.1.1 Similarity Functions for String Data	8
	2.1.2 Similarity Functions for Numeric Data	12
2.2	Record Linkage	14
2.3	Clustering	16
2.4	Blocking in Record Linkage and Clustering	18
2.5	Active Learning	19
Chapter	<b>3</b> Learnable Similarity Functions in Record Linkage	21
3.1	Learnable Similarity Functions for Strings	21

	3.1.1	Learnable Edit Distance with Affine Gaps	22
	3.1.2	Learnable Segmented Edit Distance	29
3.2	Learna	able Record Similarity	38
	3.2.1	Combining Similarity Across Fields	38
	3.2.2	Experimental Results	42
3.3	Traini	ng-Set Construction for Learning Similarity Functions	45
	3.3.1	Likely-positive Selection of Training Pairs	45
	3.3.2	Weakly-labeled Selection	50
3.4	Relate	d Work	52
3.5	Chapte	er Summary	54
Chant	or 1 Ia	arnable Similarity Functions in Sami supervised Clustering	56
	Simila	arity Functions in Clustering	56
4.1		MRE Model for Somi supervised Clustering	50
4.2	4 2 1	UMDE Model Common energy	51
	4.2.1		58
	4.2.2	Joint Probability in the HMRF Model	60
4.3	Learna	able Similarity Functions in the HMRF Model	62
	4.3.1	Parameter Priors	65
	4.3.2	Parameterized Squared Euclidean Distance	66
	4.3.3	Parameterized Cosine Distance	67
	4.3.4	Parameterized Kullback-Leibler Divergence	68
4.4	Learni	ng Similarity Functions within the HMRF-KMeans Algorithm	70
4.5	Experi	mental Results	74
	4.5.1	Datasets	75
	4.5.2	Clustering Evaluation	77
	4.5.3	Methodology	78
	4.5.4	Results and Discussion	80
4.6	Relate	d Work	91

4.7	Chapter Summary	92
Chapter	r 5 Learnable Similarity Functions in Blocking	93
5.1	Motivation	93
5.2	Adaptive Blocking Formulation	95
	5.2.1 Disjunctive blocking	98
	5.2.2 DNF Blocking	100
5.3	Algorithms	101
	5.3.1 Pairwise Training Data	101
	5.3.2 Learning Blocking Functions	101
	5.3.3 Blocking with the Learned Functions	104
5.4	Experimental Results	105
	5.4.1 Methodology and Datasets	105
	5.4.2 Results and Discussion	108
5.5	Related Work	111
5.6	Chapter Summary	112
Chapter	r 6 Future Work	113
6.1	Multi-level String Similarity Functions	113
6.2	Discriminative Pair HMMs	114
6.3	Active Learning of Similarity Functions	115
6.4	From Adaptive Blocking to Learnable Metric Mapping	116
Chapter 7 Conclusions 117		
Bibliography 12		
Vita		136

# **List of Figures**

2.1	The K-Means algorithm	17
3.1	A generative model for edit distance with affine gaps	23
3.2	Training algorithm for generative string distance with affine gaps	26
3.3	Sample coreferent records from the <i>Reasoning</i> dataset	27
3.4	Mean average precision values for field-level record linkage	29
3.5	Field linkage results for the <i>Face</i> dataset	30
3.6	Field linkage results for the <i>Constraint</i> dataset	30
3.7	Field linkage results for the <i>Reasoning</i> dataset	31
3.8	Field linkage results for the <i>Reinforcement</i> dataset	31
3.9	Segmented pair HMM	32
3.10	Sample coreferent records from the <i>Cora</i> dataset	35
3.11	Sample coreferent records from the <i>Restaurant</i> dataset	35
3.12	Field-level linkage results for the unsegmented <i>Restaurant</i> dataset	37
3.13	Field-level linkage results for the unsegmented <i>Cora</i> dataset	37
3.14	Computation of record similarity from individual field similarities	39
3.15	MARLIN overview	41
3.16	Record-level linkage results on the Cora dataset	43
3.17	Record-level linkage results on the <i>Restaurant</i> dataset	43
3.18	Mean average precision values for record-level linkage	45

3.19	Classifier comparison for record-level linkage on the Cora dataset	46
3.20	Classifier comparison for record-level linkage on the <i>Restaurant</i> dataset	46
3.21	Comparison of random and likely-positive training example selection on	
	the Restaurant dataset	48
3.22	Comparison of random and likely-positive training example selection on	
	the Cora dataset	48
3.23	Comparison of using weakly-labeled non-coreferent pairs with using ran-	
	dom labeled record pairs on the <i>Restaurant</i> dataset	51
3.24	Comparison of using weakly-labeled non-coreferent pairs with using ran-	
	dom labeled record pairs on the <i>Cora</i> dataset	51
4.1	A Hidden Markov Dandom Field for somi supervised elustering	50
4.1	A Hidden Markov Kandom Fleid för semi-supervised clustering	39
4.2	Graphical plate model of variable dependence in HMRF-based semi-supervised	
	clustering	60
4.3	The HMRF-KMEANS algorithm	71
4.4	Results for $d_{euc}$ on the <i>Iris</i> dataset $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$ $\ldots$	81
4.5	Results for $d_{euc}$ on the <i>Iris</i> dataset with full and per-cluster parameterizations	81
4.6	Results for $d_{euc}$ on the <i>Wine</i> dataset	82
4.7	Results for $d_{euc}$ on the Wine dataset with full and per-cluster parameterizations	82
4.8	Results for $d_{euc}$ on the <i>Protein</i> dataset $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	83
4.9	Results for $d_{euc}$ on the <i>Protein</i> dataset with full and per-cluster parameteri-	
	zations	83
4.10	Results for $d_{euc}$ on the <i>Ionosphere</i> dataset $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	84
4.11	Results for $d_{euc}$ on the <i>Ionosphere</i> dataset with full and per-cluster parame-	
	terizations	84
4.12	Results for $d_{euc}$ on the <i>Digits-389</i> dataset	85
4.13	Results for $d_{euc}$ on the Digits-389 dataset with full and per-cluster parame-	
	terizations	85

Results for $d_{euc}$ on the <i>Letters-IJL</i> dataset $\ldots \ldots \ldots$	86
Results for $d_{euc}$ on the Letters-IJL dataset with full and per-cluster parame-	
terizations	86
Results for $d_{\cos_A}$ on the <i>News-Different-3</i> dataset	88
Results for $d_{I_A}$ on the <i>News-Different-3</i> dataset	
Results for $d_{\cos_A}$ on the <i>News-Related-3</i> dataset $\ldots \ldots \ldots \ldots \ldots \ldots $ 8	
P Results for $d_{I_A}$ on the <i>News-Related-3</i> dataset	
Results for $d_{\cos_A}$ on the <i>News-Similar-3</i> dataset $\ldots \ldots \ldots \ldots \ldots$	90
Results for $d_{I_A}$ on the <i>News-Similar-3</i> dataset $\ldots \ldots \ldots \ldots \ldots \ldots$	90
Examples of blocking functions from different record linkage domains	94
Blocking key values for a sample record	96
Red-blue Set Cover view of disjunctive blocking	99
The algorithm for learning disjunctive blocking	102
The algorithm for learning DNF blocking	104
Blocking accuracy results for the <i>Cora</i> dataset	109
Blocking accuracy results for the <i>Addresses</i> dataset	109
	Results for $d_{euc}$ on the Letters-IJL dataset

# Chapter 1

# Introduction

### **1.1 Motivation**

Similarity functions play a central role in machine learning and data mining tasks where algorithms rely on estimates of distance between objects. Consequently, a large number of similarity functions have been developed for different data types, varying greatly in their expressiveness, mathematical properties, and assumptions. However, the notion of similarity can differ depending on the particular domain, dataset, or task at hand. Similarity between certain object features may be highly indicative of overall object similarity, while other features may be unimportant.

Many commonly used functions make the assumption that different instance features contribute equally to similarity (e.g., edit distance or Euclidean distance), while others use statistical properties of a given dataset to transform the feature space (e.g., TF-IDF weighted cosine similarity or Mahalanobis distance) (Duda, Hart, & Stork, 2001). These similarity functions make strong assumptions regarding the optimal representation of data, while they may or may not be appropriate for specific datasets and tasks. Therefore, it is desirable to *learn* similarity functions from training data to capture the correct notion of distance for a particular task in a given domain. While learning similarity functions via feature selection and feature weighting has been extensively studied in the context of classification algorithms (Aha, 1998; Wettschereck, Aha, & Mohri, 1997), use of adaptive distance measures in other tasks remains largely unexplored. In this thesis, we develop methods for adapting similarity functions to provide accurate similarity estimates in the context of the following three problems:

#### • Record Linkage

Record linkage is the general task of identifying syntactically different object descriptions objects that refer to the same underlying entity (Winkler, 2006). It has been previously studied by researchers in several areas as duplicate detection, entity resolution, object identification, and data cleaning, among several other coreferent names for this problem. Examples of record linkage include matching of coreferent bibliographic citations (Giles, Bollacker, & Lawrence, 1998), identifying the same person in different Census datasets (Winkler, 2006), and linking different offers for the same product from multiple online retailers for comparison shopping (Bilenko, Basu, & Sahami, 2005). In typical settings, performing record linkage requires two kinds of similarity functions: those that estimate similarity between individual object attributes, and those that combine such estimates to obtain overall object similarity. Object similarities are then used by matching or clustering algorithms to partition datasets into groups of equivalent objects, or perform pairwise record matching between distinct data sources.

#### • Semi-supervised Clustering

Clustering is an unsupervised learning problem in which the objective is to partition a set of objects into meaningful groups (clusters) so that objects within the same cluster are more similar to each other than to objects outside the cluster (Jain, Murty, & Flynn, 1999). In pure unsupervised settings, this objective can take on many forms depending on the semantics of "meaningful" in a specific context and on the choice of the similarity function. In semi-supervised clustering, prior information is provided to aid the grouping either in the form of objects labeled as belonging to certain categories (Basu, Banerjee, & Mooney, 2002), or in the form of pairwise constraints indicating preference for placing them in same or different clusters (Wagstaff & Cardie, 2000).

#### • Blocking

Blocking is the task of efficiently selecting a minimal subset of approximately similar object pairs from the set of all possible object pairs in a given dataset (Kelley, 1985). Because computing similarity for all object pairs is computationally costly for large datasets, to be scalable, record linkage and clustering algorithms that rely on pairwise distance estimates require blocking methods that efficiently retrieve the subset of object pairs for subsequent similarity computation. Blocking can be viewed as applying a computationally inexpensive similarity function to the entire dataset to obtain approximately similar pairs.

In these tasks, dissimilarity estimates provided by distance functions directly influence the task output and therefore can have a significant effect on performance. Thus, ensuring that employed similarity functions are appropriate for a given domain is essential for obtaining high accuracy.

This thesis presents several techniques for training similarity functions to provide accurate, domain-specific distance estimates in the context of record linkage, semi-supervised clustering and blocking. Proposed techniques are based on parameterizing traditional distance functions, such as edit distance or Euclidean distance, and learning parameter values that are appropriate for a given domain.

Learning is performed using training data in the form of pairwise supervision which consists of object pairs known to be similar or dissimilar. Such supervision has different semantics in different tasks. In record linkage, pairs of records or strings that refer to the same or different entities are known as matching and non-matching pairs (Winkler, 2006). In clustering, pairs of objects that should be placed in the same cluster or different clusters are known as must-link and cannot-link pairs, respectively (Wagstaff & Cardie, 2000). Finally, in blocking, either of the above types of supervision can be used depending on the task for which blocking is employed. Regardless of the setting, pairwise supervision is a common form of prior knowledge that is either available in many domains, or is easy to obtain via manual labeling. Our methods exploit such pairwise supervision in the three tasks listed above to learn accurate distance functions that reflect an appropriate notion of similarity for a given domain.

### **1.2 Thesis Contributions**

The goal of this thesis is proposing learnable variants of similarity functions commonly used in record linkage and clustering, developing algorithms for training such functions using pairwise supervision within these tasks, and performing experiments to study the effectiveness of the proposed methods. The contributions of the thesis are outlined below:

- We describe two learnable variants of affine-gap edit distance, a string similarity function commonly used in record linkage on string data. Based on pair Hidden Markov Models (pair HMMs) originally developed for aligning biological sequences (Durbin, Eddy, Krogh, & Mitchison, 1998), our methods lead to accuracy improvements over unlearned affine-gap edit distance and TF-IDF cosine similarity. One of the two proposed variants integrates string distance computation with string segmentation, providing a joint model for these two tasks that leads to more accurate string similarity estimates with little or no segmentation supervision. Combining learnable affine-gap edit distances across different fields using Support Vector Machines produces nearly perfect (above 0.99 F-measure) results on two standard benchmark datasets.
- We propose two strategies that facilitate efficient construction of training sets for learning similarity functions in record linkage: weakly-labeled negative and likely-positive pair selection. These techniques facilitate selecting informative training ex-

amples without the computational costs of traditional active learning methods, which allows learning accurate similarity functions using small amounts of training data.

- We describe a framework for learning similarity functions within the Hidden Markov Random Field (HMRF) model for semi-supervised clustering (Basu, Bilenko, Banerjee, & Mooney, 2006). This framework leads to embedding similarity function training within an iterative clustering algorithm, HMRF-KMEANS, which allows learning similarity functions from a combination of unlabeled data and labeled supervision in the form of same-cluster and different-cluster pairwise constraints. Our approach accommodates a number of parameterized similarity functions, leading to improved clustering accuracy on a number of text and numeric benchmark datasets.
- We develop a new framework for learning blocking functions that provides efficient and accurate selection of approximately similar object pairs for record linkage and clustering tasks. Previous work on blocking methods has relied on manually constructed blocking functions with manually tuned parameters, while our method automatically constructs blocking functions using training data that can be naturally obtained within record linkage and clustering tasks. We empirically demonstrate that our technique results in an order of magnitude increase in efficiency while maintaining high accuracy.

### **1.3** Thesis Outline

Below is a summary of the remaining chapters in the thesis:

• Chapter 2, Background. We provide the background on commonly used string and numeric similarity functions, and describe the record linkage, semi-supervised clustering and blocking tasks.

- Chapter 3, Learnable Similarity Functions in Record Linkage. We show how record linkage accuracy can be improved by using learnable string distances for individual attributes and employing Support Vector Machines to combine such distances. The chapter also discusses strategies for collecting informative training examples for training similarity functions in record linkage.
- Chapter 4, Learnable Similarity Functions in Semi-supervised Clustering. This chapter presents a summary of the HMRF framework for semi-supervised clustering and describes how it incorporates learnable similarity functions that lead to improved clustering accuracy.
- Chapter 5, Learnable Similarity Functions in Blocking. In this chapter we present a new method for automatically constructing blocking functions that efficiently select pairs of approximately similar objects for a given domain.
- **Chapter 6, Future Work.** This chapter discusses several directions for future research based on the work presented in this thesis.
- Chapter 7, Conclusions. In this chapter we review and summarize the main contributions of this thesis.

Some of the work presented here has been described in prior publications. Material presented in Chapter 3 appeared in (Bilenko & Mooney, 2003a) and (Bilenko & Mooney, 2003b), except for work described in Section 3.1.2 which has not been previously published. Material presented in Chapter 4 is a summary of work presented in a series of publications on the HMRF model for semi-supervised clustering: (Bilenko, Basu, & Mooney, 2004), (Bilenko & Basu, 2004), (Basu, Bilenko, & Mooney, 2004), and (Basu et al., 2006). Finally, an early version of the material described in Chapter 3 has appeared in (Bilenko, Kamath, & Mooney, 2006).

# **Chapter 2**

# Background

Because many data mining and machine learning algorithms require estimating similarity between objects, a number of distance functions for various data types have been developed. In this section, we provide a brief overview of several popular distance functions for text and vector-space data. We also provide background on three important problems, record linkage, clustering, and blocking, solutions for which rely on similarity estimates between observations. Finally, we introduce active learning methods that select informative training examples from a pool of unlabeled data.

Let us briefly describe the notation that we will use in the rest of this thesis. Strings are denoted by lower-case italic letters such as *s* and *t*; brackets are used for string characters and subsequences:  $s_{[i]}$  stands for *i*-th character of string *s*, while  $s_{[i:j]}$  represents the contiguous subsequence of *s* from *i*-th to *j*-th character. We use lowercase letters such as *x* and *y* for vectors, and uppercase letters such as *A* and *M* for matrices. Sets are denoted by script uppercase letters such as *X* and  $\mathcal{Y}$ .

We use the terms "distance function" and "similarity function" interchangeably when referring to binary functions that estimate degree of difference or likeness between instances.

### 2.1 Similarity functions

### 2.1.1 Similarity Functions for String Data

Techniques for calculating similarity between strings can be separated into two broad groups: sequence-based functions and vector-space-based functions. Sequence-based functions compute string similarity by viewing strings as contiguous sequences of either characters or tokens. Differences between sequences are assumed to be the result of applying edit operations that transform specific elements in one or both strings. Vector space-based functions, on other hand, do not view strings as contiguous sequences but as unordered bags of elements. Below we describe two most popular similarity functions from these groups, edit distance and TF-IDF cosine similarity. Detailed discussion of these similarity functions can be found in (Gusfield, 1997) and (Baeza-Yates & Ribeiro-Neto, 1999), respectively. For an overview of various string similarity functions proposed in the context of string matching and record linkage tasks, see (Winkler, 2006) and (Cohen, Ravikumar, & Fienberg, 2003a).

#### **Edit Distance**

Edit distance is a dissimilarity function for sequences that is widely used in many applications in natural text and speech processing (Jelinek, 1998), bioinformatics (Durbin et al., 1998), and data integration (Cohen, Ravikumar, & Fienberg, 2003b; Winkler, 2006). Classical (Levenshtein) edit distance between two strings is defined as the minimum number of edit operations (deletions, insertions, and substitutions of elements) required to transform one string into another (Levenshtein, 1966). The minimum number of such operations can be computed using dynamic programming in time equal to the product of string lengths. Edit distance can be character-based or token-based: the former assumes that every string is a sequence of characters, while the latter views strings as sequences of tokens.

For example, consider calculating character-based edit distance between strings  $s = "12 \ 8 \ Street"$  and  $t = "12 \ 8 \ th \ St$ .". There are several character edit operation sequences

of length 6 that transform s into t, implying that Levenshtein distance between s and t is 6. For example, the following six edit operations applied to s transform it into t:

1. Insert " <i>t</i> ":	"12 8 Street" $\rightarrow$ "12 8t Street"
2. Insert " <i>h</i> ":	"12 8t Street" $\rightarrow$ "12 8th Street"
3. Substitute " <i>r</i> " with ".":	"12 8th Street" $\rightarrow$ "12 8th St.eet"
4. Delete " <i>e</i> ":	"12 8th St.eet" $\rightarrow$ "12 8th St.et"
5. Delete " <i>e</i> ":	"12 8th St.et" $\rightarrow$ "12 8th St.t"
6. Delete " <i>t</i> ":	"12 8th St.t" $\rightarrow$ "12 8th St."

Wagner and Fisher (1974) generalized edit distance by allowing edit operations to have different costs. Needleman and Wunsch (1970) extended edit distance further to distinguish the cost of contiguous insertions or deletions, known as gaps, and Gotoh (1982) subsequently introduced the affine (linear) model for gap cost yielding an efficient dynamic programming algorithm for computing edit distance with gaps. The following recursions are used to compute affine-gap edit distance d(s,t) between strings *s* and *t* in O(|s||t|)computational time:

$$M(i, j) = \min \begin{cases} M(i-1, j-1) + c(s_{[i]}, t_{[j]}) \\ I_1(i-1, j-1) + c(s_{[i]}, t_{[j]}) \\ I_2(i-1, j-1) + c(s_{[i]}, t_{[j]}) \end{cases}$$

$$I_1(i, j) = \min \begin{cases} M_(i-1, j) + d + c(s_{[i]}, \varepsilon) \\ I_1(i-1, j) + e + c(s_{[i]}, \varepsilon) \end{cases}$$

$$I_2(i, j) = \min \begin{cases} M(i, j-1) + d + c(\varepsilon, t_{[j]}) \\ I_2(i, j-1) + e + c(\varepsilon, t_{[j]}) \end{cases}$$
(2.1)

 $d(s,t) = \min(M(|s|, |t|), I_1(|s|, |t|), I_2(|s|, |t|))$ 

where  $c(s_{[i]}, t_{[j]})$  is the cost of substituting (or matching) *i*-th element of *s* and *j*-th element of *t*,  $c(s_{[i]}, \varepsilon)$  and  $c(\varepsilon, t_{[j]})$  are the costs of inserting elements  $s_{[i]}$  and  $t_{[j]}$  into the first and second strings respectively (aligning this element with a gap in the other string), and *d* and *e* are the costs of starting a gap and extending it by one element. Entries (i, j) in matrices *M*,  $I_1$ , and  $I_2$  correspond to the minimal cost of an edit operation sequence between string prefixes  $s_{[1:i]}$  and  $t_{[1:j]}$  with the sequence respectively ending in a match/substitution, insertion into the first string, or insertion into the second string.

Any sequence of edit operations transforming one string into another corresponds to an *alignment* of the two strings. Alignment is a representation of the two strings obtained by inserting empty characters into the strings in place of insertions, and placing the two strings one above the other. Following is the alignment of strings s and t corresponding to the sequence of edit operations shown in the example above:

$$1 \quad 2 \quad - \quad 8 \quad \boxed{\varepsilon} \quad \boxed{\varepsilon} \quad S \quad t \quad r \quad e \quad e \quad t$$

$$1 \quad 2 \quad - \quad 8 \quad t \quad h \quad - \quad S \quad t \quad \cdot \quad \boxed{\varepsilon} \quad \boxed{\varepsilon} \quad \boxed{\varepsilon} \quad \boxed{\varepsilon} \quad (2.2)$$

This representation shows that the sequence of edit operations for any alignment can be viewed as an a production of the two strings in parallel by emitting elements from either one or both strings simultaneously. This view will be central in the development of learnable affine-gap edit distance in Chapter 3.

#### Jaccard and TF-IDF Cosine Similarity

While sequence-based string similarity functions work well for estimating distance between shorter strings, they become too computationally expensive and less accurate for longer strings. For example, when differences between equivalent strings are due to long-range transpositions of multiple words, sequence-based similarity functions assign high cost to non-aligned string segments, resulting in low similarity scores for strings that share many common words. At the same time, computing string edit distance becomes computationally prohibitive for larger strings such as text documents on the Web because its computational complexity is quadratic in string size.

The vector-space model of text avoids these problems by viewing strings as "bags of tokens" and disregarding the order in which the tokens occur in the strings (Salton & McGill, 1983). Jaccard similarity can then be used as the simplest method for computing likeness as the proportion of tokens shared by both strings. If strings *s* and *t* are represented by sets of tokens S and T, Jaccard similarity is:

$$sim_{Jaccard}(s,t) = \frac{|S \cap T|}{|S \cup T|}$$
(2.3)

The primary problem with Jaccard similarity is that it does not take into account the relative importance of different tokens. Tokens that occur frequently in a given string should have higher contribution to similarity than those that occur few times, as should those tokens that are rare among the set of strings under consideration. The Term Frequency-Inverse Document Frequency (TF-IDF) weighting scheme achieves this by associating a weight  $w_{v_i,s} = \frac{N(v_i,s)}{\max_{v_j \in s} N(v_j,s)} \cdot \log \frac{N}{N(v_i)}$  with every token  $v_i$  from string *s*, where  $N(v_i, s)$  is the number of times  $v_i$  occurs in *s* (term frequency), *N* is the number of strings in the overall corpus under consideration, and  $N(v_i)$  is the number of strings in the corpus that include  $v_i$ (document frequency).

Given a corpus of strings that yields the set  $\mathcal{V}$  of distinct tokens after tokenization, a string *s* can be represented as a  $|\mathcal{V}|$ -dimensional vector of weights, every non-zero component of which corresponds to a token present in *s*. TF-IDF cosine similarity between two strings is defined as the cosine of the angle between their vector representations:

$$sim_{TF-IDF}(s,t) = \frac{w_s^T w_t}{\|w_s\| \|w_t\|} = \frac{\sum_{v_i \in \mathcal{V}} w_{s,v_i} w_{t,v_i}}{\sqrt{\sum_{s_i \in \mathcal{S}} w_{s,s_i}^2} \cdot \sqrt{\sum_{t_i \in \mathcal{T}} w_{t,t_i}^2}}$$
(2.4)

With the help of appropriate inverted index data structures, TF-IDF cosine similarity is computationally efficient due to high sparsity of most vectors, and provides a reasonable off-the-shelf metric for long strings and text documents. Tokenization is typically performed by treating each individual word of certain minimum length as a separate token, usually excluding a fixed set of functional "stop words" and optionally stemming tokens to their roots (Baeza-Yates & Ribeiro-Neto, 1999). An alternative tokenization scheme is known as n-grams: it relies on using all overlapping contiguous character subsequences of length n as tokens.

#### 2.1.2 Similarity Functions for Numeric Data

#### **Euclidean and Mahalanobis distances**

For data represented by vectors in Euclidean space, the Minkowski family of metrics, also known as the  $L_k$  norms, includes most commonly used similarity measures for objects described by *d*-dimensional vectors (Duda et al., 2001):

$$L_k(x_i, x_j) = \left(\sum_{l=1}^d |x_{il} - x_{jl}|^k\right)^{1/k}$$
(2.5)

The  $L_2$  norm, commonly known as Euclidean distance, is frequently used for lowdimensional vector data. Its popularity is due to a number of factors:

- Intuitive simplicity: the *L*<sub>2</sub> norm corresponds to straight-line distance between points in Euclidean space;
- Invariance to rotation or translation in feature space;
- Mathematical metric properties: non-negativity  $(L_2(x_i, x_j) \ge 0))$ , reflexivity  $(L_2(x_i, x_j) = 0$  iff  $x_i = x_j)$ , symmetry  $(L_2(x_i, x_j) = L_2(x_j, x_i))$ , and triangle inequality  $(L_2(x_i, x_j) + L_2(x_j, x_k) \ge L_2(x_i, x_k))$ , that allow using it in many algorithms that rely on metric assumptions.

If distance is computed among points of a given dataset, Mahalanobis distance is an extension of Euclidean distance that takes into account the data mean as well as variance

of each dimension and correlations between the different dimensions, which are estimated from the dataset. Given a set of observation vectors  $\{x_1, ..., x_n\}$ , Mahalanobis distance is defined as:

$$d_{Mah}(x_i, x_j) = ((x_i - x_j)^T \Sigma^{-1} (x_i - x_j))^{1/2}$$
(2.6)

where  $\Sigma^{-1}$  is the inverse of the covariance matrix  $\Sigma = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu) (x_i - \mu)^T$ , and  $\mu = \frac{1}{n} \sum_{i=1}^{n} x_i$  is the data mean.

Essentially, Mahalanobis distance attempts to give each dimension equal weight when computing distance by scaling its contribution proportionally to variance, while taking into account co-variances between the dimensions.

#### **Cosine Similarity**

Minkowski metrics including Euclidean distance suffer from the *curse of dimensionality* when they are applied to high-dimensional data (Friedman, 1997). As the dimensionality of the Euclidean space increases, sparsity of observations increases exponentially with the number of dimensions, which leads to observations becoming equidistant in terms of Euclidean distance. Cosine similarity, or normalized dot product, has been widely used as an alternative similarity function for high-dimensional data (Duda et al., 2001):

$$Sim_{cos}(x,y) = \frac{x^T y}{\|x\| \|y\|} = \frac{\sum_{i=1}^d x_i \cdot y_i}{\sqrt{\sum_{i=1}^d x_i^2} \cdot \sqrt{\sum_{i=1}^d y_i^2}}$$
(2.7)

If applied to normalized vectors, cosine similarity obeys metric properties when converted to distance by negating it from 1. In general, however, it is not a metric in the mathematical sense, and it is not invariant to translations and linear transformations.

#### Information-theoretic Measures

In certain domains, data can be described by probability distributions, e.g., text documents can be represented as probability distributions over words generated by a multinomial model (Pereira, Tishby, & Lee, 1993). Kullback-Leibler (KL) divergence, also known as relative entropy, is a widely used distance measure for such data:

$$d_{KL}(x_i, x_j) = \sum_{m=1}^{d} x_{im} \log \frac{x_{im}}{x_{jm}}$$
(2.8)

where  $x_i$  and  $x_j$  are instances described by probability distributions over d events:  $\sum_{m=1}^{d} x_{im} = \sum_{m=1}^{d} x_{jm} = 1$ . Note that KL divergence is not symmetric:  $d_{KL}(x_i, x_j) \neq d_{KL}(x_i, x_j)$  for any  $x_i \neq x_j$ . In domains where a symmetrical distance function is needed, Jensen-Shannon divergence, also known as KL divergence to the mean, is used:

$$d_{JS}(x_i, x_j) = \frac{1}{2} \left( d_{KL}(x_i, \frac{x_i + x_j}{2}) + d_{KL}(x_j, \frac{x_i + x_j}{2}) \right)$$
(2.9)

Kullback-Leibler divergence is widely used in information theory (Cover & Thomas, 1991), where it is interpreted as the expected extra length of a message sampled from distribution  $x_i$  encoded using a coding scheme that is optimal for distribution  $x_i$ .

### 2.2 Record Linkage

As defined in Chapter 1, the goal of record linkage is identifying instances that differ syntactically yet refer to the same underlying object. Matching of coreferent bibliographic citations and identifying multiple variants of a person's name or address in medical, customer, or census databases are instances of this problem. A number of researchers in different communities have studied variants of record linkage tasks : after being introduced in the context of matching medical records by Newcombe, Kennedy, Axford, and James (1959), it was investigated under a number of names including merge/purge (Hernández & Stolfo, 1995), heterogeneous database integration (Cohen, 1998), hardening soft databases (Cohen, Kautz, & McAllester, 2000), reference matching (McCallum, Nigam, & Ungar, 2000), de-duplication (Sarawagi & Bhamidipaty, 2002; Bhattacharya & Getoor, 2004), fuzzy duplicate elimination (Ananthakrishna, Chaudhuri, & Ganti, 2002; Chaudhuri, Ganjam, Ganti, & Motwani, 2003), entity-name clustering and matching (Cohen & Richman, 2002), identity uncertainty (Pasula, Marthi, Milch, Russell, & Shpitser, 2003; McCallum & Wellner, 2004a), object consolidation (Michalowski, Thakkar, & Knoblock, 2003), robust reading (Li, Morie, & Roth, 2004), reference reconciliation (Dong, Halevy, & Madhavan, 2005), object identification (Singla & Domingos, 2005), and entity resolution (Bhattacharya & Getoor, 2006).

The seminal work of Fellegi and Sunter (1969) described several key ideas that have been used or re-discovered by most record linkage researchers, including combining similarity estimates across multiple fields, using blocking to reduce the set of candidate record pairs under consideration, and using a similarity threshold to separate the coreferent and non-coreferent object pairs. Fellegi and Sunter (1969) considered record linkage in an unsupervised setting where no examples of coreferent and non-coreferent pairs are available. In this setting, several methods have been proposed that rely on learning probabilistic models with latent variables that encode the matching decisions (Winkler, 1993; Ravikumar & Cohen, 2004). In the past decade, a number of researchers have considered record linkage settings where pairwise supervision is available, allowing the application of such classification techniques as decision trees (Elfeky, Elmagarmid, & Verykios, 2002; Tejada, Knoblock, & Minton, 2001), logistic regression (Cohen & Richman, 2002), Bayesian networks (Winkler, 2002), and Support Vector Machines (Bilenko & Mooney, 2003a; Cohen et al., 2003a; Minton, Nanjo, Knoblock, Michalowski, & Michelson, 2005) to obtain record-level distance functions that combine the field-level similarities. These methods treat individual field similarities as features and train a classifier to distinguish between coreferent and non-coreferent records, using the confidence of the classifier's prediction as the similarity estimate.

The majority of solutions for record linkage treat it as a modular problem that is solved in multiple stages. In the first stage, blocking is performed to obtain a set of candidate record pairs to be investigated for co-reference, since the computational cost of computing pairwise similarities between all pairs of records in a large database is often prohibitive; see Section 2.4 for discussion of blocking. In the second stage, similarity is computed between individual fields of candidate record pairs. In the final linkage stage, similarity is computed between candidate pairs, and highly similar records are labeled as matches that describe the same entity. Linkage can be performed either via *pairwise* inference where decisions for the different candidate pairs are made independently, or via *collective* inference over all candidate record pairs (Pasula et al., 2003; Wellner, McCallum, Peng, & Hay, 2004; Singla & Domingos, 2005).

### 2.3 Clustering

Clustering is typically defined as the problem of partitioning a dataset into disjoint groups so that observations belonging to the same cluster are similar, while observations belonging to different clusters are dissimilar. Clustering has been widely studied for several decades, and a great variety of algorithms for clustering have been proposed (Jain et al., 1999). Several large groups of clustering algorithms can be distinguished that include hierarchical clustering methods that attempt to create a hierarchy of data partitions (Kaufman & Rousseeuw, 1990), partitional clustering methods that separate instances into disjoint clusters (Karypis & Kumar, 1998; Shi & Malik, 2000; Strehl, 2002; Banerjee, Merugu, Dhilon, & Ghosh, 2005b), and overlapping clustering techniques that allow instances to belong to multiple clusters (Segal, Battle, & Koller, 2003; Banerjee, Krumpelman, Basu, Mooney, & Ghosh, 2005c).

Traditionally, clustering has been viewed as a form of unsupervised learning, since no class labels for the data are provided. In *semi-supervised clustering*, supervision from a user is incorporated in the form of class labels or pairwise constraints on objects which can be used to initialize clusters, guide the clustering process, and improve the clustering algorithm parameters (Basu, 2005).

Work presented in Chapter 4 is based on K-Means, a widely used clustering algo-

rithm that performs iterative relocation of cluster centroids to locally minimize the total distance between the data points and the centroids. Given a set of data points  $\mathcal{X} = \{x_i\}_{i=1}^N, x_i \in \mathbb{R}^m$ , let  $\{\mu_h\}_{h=1}^K$  represent the *K* cluster centroids, and  $y_i$  be the cluster assignment of a point  $x_i$ , where  $y_i \in \mathcal{Y}$  and  $\mathcal{Y} = \{1, \dots, K\}$ . The Euclidean K-Means algorithm creates *K* disjoint subsets of  $\mathcal{X}, \{\mathcal{X}_l\}_{l=1}^K$ , whose union is  $\mathcal{X}$ , so that the following objective function is (locally) minimized:

$$J_{kmeans}(\mathcal{X}, \mathcal{Y}) = \sum_{x_i \in \mathcal{X}} \|x_i - \mu_{y_i}\|^2$$
(2.10)

Intuitively, this objective function measures the tightness of each cluster as the sum of squared Euclidean distances between every point in the cluster and the centroid. Figure 2.1 presents the pseudocode for the algorithm.

Algorithm: K-MEANS Input: Set of data points  $X = \{x_i\}_{i=1}^n, x_i \in \mathbb{R}^d$ , number of clusters KOutput: Disjoint K-partitioning  $\{X_h\}_{h=1}^K$  of X such that objective function  $J_{kmeans}$  is optimized Method: 1. Initialize clusters: Initial centroids  $\{\mu_h^{(0)}\}_{h=1}^K$  are selected at random 2. Repeat until *convergence* 2a. assign\_cluster: Assign each data point  $x_i$  to the cluster  $h^*$  (i.e. set  $X_{h^*}^{(t+1)}$ ), where  $h^* = \arg\min_h ||x_i - \mu_h^{(t)}||^2$ 2b. estimate\_means:  $\mu_h^{(t+1)} \leftarrow \frac{1}{|X_h^{(t+1)}|} \sum_{x_i \in X_h^{(t+1)}} x_i$ 2c.  $t \leftarrow (t+1)$ 

Figure 2.1: The K-Means algorithm

Recently, it has been shown that K-Means-style algorithms can be derived based on a number of dissimilarity functions including directional measures such as cosine similarity (Banerjee, Dhillon, Ghosh, & Sra, 2005a) and a large class of functions known as Bregman divergences, which include squared Euclidean distance and KL-divergence (Banerjee et al., 2005b).

## 2.4 Blocking in Record Linkage and Clustering

Because the number of similarity computations grows quadratically with the size of the input dataset, scaling up to large datasets is problematic for tasks that require similarities between all instance pairs. Additionally, even for small datasets, estimation of the full similarity matrix can be difficult if computationally costly similarity functions, distance metrics or kernels are used. At the same time, in many tasks, the majority of similarity computations are unnecessary because most instance pairs are highly dissimilar and have no influence on the task output. Avoiding the unnecessary computations results in a sparse similarity matrix, and a number of algorithms become practical for large datasets when provided with sparse similarity matrices, e.g. the collective inference algorithms for record linkage (Pasula et al., 2003; McCallum & Wellner, 2004b; Singla & Domingos, 2005).

Blocking methods efficiently select a subset of instance pairs for subsequent similarity computation, ignoring the remaining pairs as highly dissimilar and therefore irrelevant. A number of blocking algorithms have been proposed by researchers in recent years, all of which rely on a manually tuned set of predicates or parameters (Fellegi & Sunter, 1969; Kelley, 1985; Jaro, 1989; Hernández & Stolfo, 1995; McCallum et al., 2000; Baxter, Christen, & Churches, 2003; Chaudhuri et al., 2003; Jin, Li, & Mehrotra, 2003; Winkler, 2005).

Key-based blocking methods form blocks by applying some unary predicate to each record and assigning all records that return the same value (key) to the same block (Kelley, 1985; Jaro, 1989; Winkler, 2005). For example, such predicates as *Same Zipcode* or *Same 3-character Prefix of Surname* could be used to perform key-based blocking in a name-address database, resulting in blocks that contain records with the same value of the *Zipcode* attribute and the same first three characters of the *Surname* attribute, respectively.

Another popular blocking technique is the sorted neighborhood method proposed by Hernández and Stolfo (1995). This method forms blocks by sorting the records in a database using lexicographic criteria and selecting all records that lie within a window of fixed size. Multiple sorting passes are performed to increase coverage.

The canopies blocking algorithm of McCallum et al. (2000) relies on a similarity function that allows efficient retrieval of all records within a certain distance threshold from a randomly chosen record. Blocks are formed by randomly selecting a "canopy center" record and retrieving all records that are similar to the chosen record within a certain ("loose") threshold. Records that are closer than a "tight" threshold are removed from the set of possible canopy centers, which is initialized with all records in the dataset. This process is repeated iteratively, resulting in formation of blocks selected around the canopy centers. Inverted index-based similarity functions such as Jaccard or TF-IDF cosine similarity are typically used with the canopies method as they allow fast selection of nearest neighbors based on co-occurring tokens. Inverted indices are also used in the blocking method of Chaudhuri et al. (2003), who proposed using indices based on character *n*-grams for efficient selection of candidate record pairs.

Recently, Jin et al. (2003) proposed a blocking method based on mapping database records to a low-dimensional metric space based on string values of individual attributes. While this method can be used with arbitrary similarity functions, it is computationally expensive compared to the sorting and index-based methods.

### 2.5 Active Learning

When training examples are selected for a learning task at random, they may be suboptimal in the sense that they do not lead to a maximally attainable improvement in performance. *Active learning* methods attempt to identify those examples that lead to maximal accuracy improvements when added to the training set (Lewis & Catlett, 1994; Cohn, Ghahramani, & Jordan, 1996; Tong, 2001). During each round of active learning, the example that is estimated to improve performance the most when added to the training set is identified and labeled. The system is then re-trained on the training set including the newly added labeled example.

Three broad classes of active learning methods exist: (1) uncertainty sampling techniques (Lewis & Catlett, 1994) attempt to identify examples for which the learning algorithm is least certain in its prediction; (2) query-by-committee methods (Seung, Opper, & Sompolinsky, 1992) utilize a committee of learners and use disagreement between committee members as a measure of training examples' informativeness; (3) estimation of error reduction techniques (Lindenbaum, Markovitch, & Rusakov, 1999; Roy & McCallum, 2001) select examples which, when labeled, lead to greatest reduction in error by minimizing prediction variance.

Active learning was shown to be a successful strategy for improving performance using small amounts of training data on a number of tasks, including classification (Cohn et al., 1996), clustering (Hofmann & Buhmann, 1998; Basu, Banerjee, & Mooney, 2004), and record linkage (Sarawagi & Bhamidipaty, 2002; Tejada, Knoblock, & Minton, 2002).
# **Chapter 3**

# Learnable Similarity Functions in Record Linkage

In this chapter, we describe the use of learnable similarity functions in record linkage, where they improve the accuracy of distance estimates in two tasks: computing similarity of string values between individual record fields, and combining such similarities across multiple fields to obtain overall record similarity. At the field level, two adaptive variants of edit distance are described that allow learning the costs of string transformations to reflect their relative importance in a particular domain. At the record level, we employ Support Vector Machines, a powerful discriminative classifier, to distinguish between pairs of similar and dissimilar records. We also propose two strategies for automatically selecting informative pairwise training examples. These strategies do not require the human effort needed by active learning methods, yet vastly outperform random pair selection.

# 3.1 Learnable Similarity Functions for Strings

In typical linkage applications, individual record fields are represented by short string values whose length does not exceed several dozen characters or tokens. For such strings, differences between coreferent values frequently arise due to local string transformations at either character or token level, e.g., misspellings, abbreviations, insertions, and deletions. To capture such differences, similarity functions must estimate the total cost associated with performing these transformations on string values.

As described in Section 2.1.1, edit distance estimates string dissimilarity by computing the cost of a minimal sequence of edit operations required to transform one string into another. However, the importance of different edit operations varies from domain to domain. For example, a digit substitution makes a big difference in a street address since it effectively changes the house or apartment number, while a single letter substitution is semantically insignificant because it is more likely to be caused by a typographic error or an abbreviation. For token-level edit distance, some tokens are unimportant and therefore their insertion cost should be low, e.g., for token *St.* in street addresses.

Ability to vary the gap cost is a significant advantage of affine-gap edit distance over Levenshtein edit distance, which penalizes all insertions independently (Bilenko & Mooney, 2002). Frequency and length of gaps in string alignments also vary from domain to domain. For example, during linkage of coreferent bibliographic citations, gaps are common for the *author* field where names are often abbreviated, yet rare for the *title* field which is typically unchanged between citations to the same paper.

Therefore, adapting affine-gap edit distance to a particular domain requires learning the costs for different edit operations and the costs of gaps. In the following subsections, we present two methods that perform such of edit distance parameters using a corpus of coreferent string pairs from a given domain.

#### 3.1.1 Learnable Edit Distance with Affine Gaps

#### The Pair HMM Model

We propose learning the costs of edit distance parameters using a three-state pair HMM shown in Figure 3.1. It extends the one-state model used by Ristad and Yianilos (1998)



Figure 3.1: A generative model for edit distance with affine gaps

to learn parameters of Levenshtein edit distance, and is analogous to models proposed in Durbin et al. (1998) for scoring alignments of biological sequences.

For any pair of strings, the model can generate all possible alignments between them as sequences of state transitions and edit operation emissions, where emissions correspond to productions of elements of the two strings in parallel, including gaps. Each possible alignment is associated with the probability of observing the corresponding sequence of transitions and emissions.

The three states of the model generate gaps in the alignment in states  $I_1$  and  $I_2$ , and generate matches and substitutions in state M. Transitions between state M and states  $I_1$ and  $I_2$  in the pair HMM correspond to starting a gap in the deterministic affine-gap edit distance model, while self-transitions in states  $I_1$  and  $I_2$  model gap extensions. Probabilities of these transitions,  $\sigma$  and  $\delta$ , correspond to gap opening and extension costs, while probabilities  $\mu$ ,  $\gamma_M$  and  $\gamma_I$  correspond to the relative frequency of continued matching, gap ending, and observing adjacent gaps (these transitions have no direct analog in the deterministic model).<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>Traditional edit distance algorithms as well as pair HMMs described by Durbin et al. (1998) also disallow gaps in the two strings to be contiguous. This restriction corresponds to prohibiting transitions between states  $I_1$  and  $I_2$ , but in the record linkage domain it is unnecessary since the two strings may have parallel non-matching

Emissions in the pair HMM correspond to individual edit operations that generate both strings in parallel. Given  $\mathcal{R}^* = \mathcal{R} \cup \{\varepsilon\}$ , the symbol alphabet extended with the special "gap" character  $\varepsilon$ , the full set of edit operations is  $\mathcal{E} = \mathcal{E}_M \cup \mathcal{E}_{I_1} \cup \mathcal{E}_{I_2}$ , where  $\mathcal{E}_M = \{\langle a, b \rangle :$  $a, b \in \mathcal{R}\}$  is the set of all substitution and matching operations, while  $\mathcal{E}_{I_1} = \{\langle a, \varepsilon \rangle : a \in \mathcal{R}\}$  and  $\mathcal{E}_{I_2} = \{\langle \varepsilon, a \rangle : a \in \mathcal{R}\}$  are the insertions into the first and into the second strings respectively. Each state associates its set of emissions with a probability distribution. Thus, emission probabilities in the pair HMM,  $\mathcal{P}_M = \{p(e) : e \in \mathcal{E}_M\}$ ,  $\mathcal{P}_{I_1} = \{p(e) : e \in \mathcal{E}_{I_1}\}$ , and  $\mathcal{P}_{I_2} = \{p(e) : e \in \mathcal{E}_{I_2}\}$ , correspond to costs of individual edit operations in the deterministic model. Edit operations with higher probabilities produce character pairs that are likely to be aligned in a given domain, e.g., substitution  $\langle /, - \rangle$  for phone numbers or deletion  $\langle ., \varepsilon \rangle$ for addresses. For each state in the pair HMM, there is an associated probability of starting or ending the string alignment in that state, corresponding to the frequency of observing alignments with gaps at the beginning or at the end.

Because in record linkage applications the order of two strings is unimportant, several parameters in the model are tied to make alignments symmetrical with respect to the two strings. Tied parameters include probabilities of transitions entering and exiting the insertion states:  $\sigma$ ,  $\gamma_M$ ,  $\gamma_I$ , and  $\delta$ ; emission probabilities for the insertion states:  $p(\langle a, \varepsilon \rangle) = p(\langle \varepsilon, a \rangle)$ , and emissions of substitutions  $p(\langle a, b \rangle) = p(\langle b, a \rangle)$ .

Two methods can be used for computing edit distance using a trained pair HMM. The Viterbi algorithm computes the highest-probability alignment of two strings, while the forward (or backward) algorithm computes the total probability of observing all possible alignments of the strings, which can be beneficial if several high-probability alignments exist (Rabiner, 1989). If performed in log-space, the algorithms are analogous to the deterministic edit distance computation shown in Eq. (2.1), with the negative logarithms of probabilities replacing the corresponding costs. The three matrices of the deterministic affine-gap edit distance described in Section 2.1.1 correspond to dynamic programming regions.

matrices computed by the Viterbi, forward and backward algorithms. Each entry (i, j) in the dynamic programming matrices for states M,  $I_1$ , and  $I_2$  contains the forward, backward, or Viterbi probability for aligning prefixes  $x_{[1:i]} x_{[1:j]}$  and ending the transition sequence(s) in the corresponding state.

#### Training

Given a training set of *N* coreferent string pairs  $\mathcal{D} = \{(x_i^{(1)}, x_i^{(2)})\}$ , the transition and emission probabilities in the model can be learned using a variant of the Baum-Welch algorithm outlined in Figure 3.2, which is an Expectation-Maximization procedure for learning parameters of HMMs (Rabiner, 1989); Ristad and Yianilos (1998) used an analogous algorithm for training their one-state model for Levenshtein distance. The training procedure iterates between two steps, expectation (E-step) and maximization (M-step), converging to a (local) maximum of the log-likelihood of training data  $\mathcal{L} = \sum_{i=1..N} \log p_{\Theta}(x_i^{(1)}, x_i^{(2)})$ , where  $\Theta = \{\mu, \delta, \sigma, \gamma_M, \gamma_I, \mathcal{P}_M, \mathcal{P}_{I_1}, \mathcal{P}_{I_2}\}$  is the set of emission and transition probabilities being learned. In the E-step, the forward and backward matrices are computed for every training pair to accumulate the expected number of transitions and emissions given current parameter values (Rabiner, 1989). In the M-step, parameters  $\Theta$  are updated by re-normalizing the expectations of transition and emission probabilities accumulated in the E-step.

Once trained, the model can be used for estimating similarity between pairs of strings by using the forward algorithm to compute the probability of generating all possible string alignments. To prevent numerical underflow for long strings, this computation should be performed in log-space.

Modeling edit distances with pair HMMs has an intrinsic drawback: because probability of generating a string pair decreases with string length, alignments of longer strings have lower probabilities that alignments of shorter strings. This problem is alleviated by using length-corrected distance  $d(x,y) = -\log p(x,y)^{1/(|x|+|y|)}$ , which is equivalent to scaling deterministic edit distance by the sum of string lengths. Furthermore, the standard



Figure 3.2: Training algorithm for generative string distance with affine gaps

pHMM must normalize the probability of the exact match operations against the probabilities of substitutions, and normalize  $\mu$ , the probability of the self-transition in state M, against 2 $\delta$ , the probabilities of starting a gap in either string. These normalizations imply that even the perfect matching of a string to itself has a less-than-1 probability, which is counter-intuitive, yet unavoidable within the pair HMM generative framework. However, we have found that setting the costs (log-probabilities) of *M*-state self-transitions and match emissions to 0 leads to improved empirical results, although the pair HMM model does not provide a principled way of encoding this intuition.

#### **Experimental Evaluation**

We evaluated the proposed model for learnable affine-gap edit distance on four datasets. *Face,Constraint, Reasoning,* and *Reinforcement* are single-field datasets containing unsegmented citations to computer science papers in corresponding areas from the *Citeseer* digital library (Giles et al., 1998). *Face* contains 349 citations to 242 papers, *Constraint* contains 295 citations to 199 papers, and *Reasoning* contains 514 citation records that represent 196 unique papers, and *Reinforcement* contains 406 citations to 148 papers. Figure 3.3 presents sample coreferent records from one of the datasets.

Every dataset was randomly split into 2 folds for cross-validation during each experimental run. A larger number of folds is impractical since it would result in fewer coreferent pairs per fold. To create the folds, coreferent records were grouped together, and the resulting clusters were randomly assigned to the folds. All results are reported over 10 random splits, where for each split the two folds were used alternately for training and testing.

During each trial, learnable edit distance is trained as described above using randomly sampled pairs of coreferent strings from the training fold. After training, edit distance is computed between all pairs of strings in the testing fold. Then, pairs are iteratively labeled as coreferent in order of decreasing similarity. After labeling of each successive string pair, accuracy is evaluated using pairwise precision and recall, which are computed as follows:

Figure 3.3: Sample coreferent records from the *Reasoning* dataset

L. P. Kaelbling. An architecture for intelligent reactive systems. In Reasoning
About Actions and Plans: Proceedings of the 1986 Workshop. Morgan Kaufmann, 1986
Kaelbling, L. P., 1987. An architecture for intelligent reactive systems. In
M. P. Georgeff & A. L. Lansky, eds., Reasoning about Actions and Plans, Morgan
Kaufmann, Los Altos, CA, 395 410

 $precision = \frac{\# of CorrectCoreferentPairs}{\# of LabeledPairs}$  $recall = \frac{\# of CorrectCoreferentPairs}{\# of TrueCoreferentPairs}$ 

We also compute mean average precision (MAP), defined as follows:

$$MAP = \frac{1}{n} \sum_{i=1}^{n} precision(i)$$
(3.1)

where *n* is the number of true coreferent pairs in the dataset, and precision(i) is the pairwise precision computed after correctly labeling *i*-th coreferent pair. These measures evaluate how well a similarity function distinguishes between coreferent and non-coreferent string pairs: a perfect string distance would assign higher similarity to all coreferent pairs than to any non-coreferent pair, achieving 1.0 on all metrics. On the precision-recall curve, precision at any recall level corresponds to the fraction of pairs above a certain similarity threshold that are coreferent, while lowering the threshold results in progressive identification of more truly coreferent pairs. For averaging the results across multiple trials, precision is interpolated at fixed recall levels following the standard methodology from information retrieval (Baeza-Yates & Ribeiro-Neto, 1999).

To evaluate the usefulness of adapting affine-gap string edit distance to a specific domain, we compare the pair HMM-based learnable affine-gap edit distance with its fixed-cost equivalent on the task of identifying equivalent field values, as well as with classic Levenshtein distance. The following results are presented:

- PHMM LEARNABLE ED: learnable affine-gap edit distance based on characters trained as described above using the EM algorithm shown in Fig.3.2;
- UNLEARNED ED: fixed-cost affine-gap edit distance (Gusfield, 1997) with a substitution cost of 5, gap opening cost of 5, gap extension cost of 1, and match cost of -5,

Distance metric	Face	Constraint	Reasoning	Reinforcement
pHMM Learnable edit distance	0.960	0.968	0.955	0.961
Unlearned edit distance	0.956	0.956	0.946	0.952
Levenshtein edit distance	0.901	0.874	0.892	0.899

Figure 3.4: Mean average precision values for field-level record linkage

which are parameters previously suggested by (Monge & Elkan, 1996);

• LEVENSHTEIN: classic Levenshtein distance described in Section 2.1.1.

Precision-recall curves for the four datasets are shown on Figures 3.5-3.8. These results are summarized in Figure 3.4. Each entry contains the mean average precision over the 20 evaluated folds. Improvements of the learnable edit distance over the fixed-cost variant are significant at the 0.05 level using a two-tailed paired t-test for all datasets. These results demonstrate that learned affine-gap edit distance outperforms its deterministic equivalent in identifying coreferent values in individual fields, which in turn is significantly more accurate than Levenshtein distance.

#### **3.1.2** Learnable Segmented Edit Distance

Affine-gap edit distance and the corresponding pair HMM model described in Section 3.1.1 treat strings as homogeneous entities. In domains where strings are composed of multiple fields, such as bibliographic citations, ignoring their internal structure disregards the differences between edit distance parameters in appropriate models for the fields, while some string transformations may be frequent in one field, but rare in another. For affine-gap edit distance derived from a pair HMM, rarity of certain operations (e.g., rarity of gaps for *ti*-*tle* values) corresponds to a lower value of  $\sigma$ , probability of the gap opening transition. Training individual pair HMM distances for every field allows making such distinctions. Therefore, segmenting strings into individual fields can improve the accuracy of similarity computations, and in domains where accurate segmentation is available, or original data is described by multiple fields, combining multiple field-specific distances was shown to be







Figure 3.6: Field linkage results for the Constraint dataset



Figure 3.7: Field linkage results for the Reasoning dataset



Figure 3.8: Field linkage results for the Reinforcement dataset

effective for the record linkage task, as results in Section 3.2 will show.

However, in domains where supervision in the form of segmented strings for training an information extraction system is limited, field values cannot be extracted reliably. Segmentation mistakes lead to erroneous field-level estimates of similarity, combining which may produce worse results than utilizing a single string similarity function.

#### **Segmented Pair HMM**

We propose a new type of pair HMM, the segmented pair HMM (spHMM), that overcomes the above limitations by combining segmentation and edit distance computation within a single framework. A sample spHMM is shown in Figure 3.9. It can be viewed as an interconnected sequence of pair HMMs, where the emission and transition probabilities within each pair HMM are trained for a particular field, while probabilities of transitions between the pair HMMs capture the field structure of the strings. The model generates string matchings by emitting alignments of individual fields in corresponding components, transitioning between them at segment boundaries in both strings simultaneously.



Figure 3.9: Segmented pair HMM

As in regular pair HMMs, edit distance between two strings *x* and *y* in the spHMM is computed as the negative logarithm of the probability of generating the string pair over all possible alignments,  $d(x, y) = -\log p(x, y)$ , which can be computed using the standard

forward (or backward) algorithm. This allows aggregating alignment probabilities over the different possible segmentations of the two strings, which is not achievable if segmentation and matching are performed in isolation. The obtained distance value is length-corrected to avoid penalizing longer strings  $d(x, y) = -\log p(x, y)^{(|x|+|y|)}$ .

#### Training

As with the learnable affine-gap edit distance without segmentation described in Section 3.1.1, transition and emission probabilities of the spHMM are learned using a training set  $\mathcal{D} = \{(x_i^{(1)}, x_i^{(2)})\}\{(x_i, y_i)\}_{i=1}^N$  consisting of *N* string pairs. Training is performed using an extension of the Expectation-Maximization (EM) procedure shown in Figure 3.2 that learns an extended set of emission and transition probabilities for all *k* pair HMMs in the spHMM:  $\Theta = \{\mu^{(i)}, \delta^{(i)}, \sigma^{(i)}, \gamma_M^{(i)}, \gamma_I^{(i)}, \tau_1^{(i)}, \dots, \tau_k^{(i)}, \mathcal{P}_M^{(i)}, \mathcal{P}_{I_1}^{(i)}, \mathcal{P}_{I_2}^{(i)}\}_{i=1}^k$ . Probabilities of transitions between pair HMMs,  $\{\tau_1^{(i)}, \dots, \tau_k^{(i)}\}_{i=1}^k$ , are learned by decomposing them into transitions outgoing from individual states  $M^{(i)}, I_1^{(i)}, I_2^{(i)}$  into any other state outside the *i*-th component, and tying the parameters over all such transitions for any two pair HMMs.

Training can incorporate any combination of supervision used for segmentation and string similarity computation tasks. There are three types of training data that may be available:

(i) pairs of coreferent segmented strings, e.g.

	author		title		othe	r		year
	M.J. Kearns. The Computational Complexity of Machine Learning. M			AIT Press, Can	nbridge, I	MA	(1990).	
ĺ	Michael Kearns.		The Computational Complexity of Machine Learning	g.	MIT Press,	1990.		
1	author		title		other	year		

(ii) pairs of coreferent unsegmented strings, e.g.

M. Kearns, R. Schapire and L. Sellie, Towards efficient agnostic learning. COLT, 1992 Kearns, M., Schapire, R., and Sellie, L. (1992) Toward efficient agnostic learning. In Proc. 5th Ann. Workshop on Computational Learning Theory. Pittsburgh, PA: Morgan Kaufmann.

(iii) individual segmented strings, e.g.

author	year	title	venue	other	
Freund, Y.	(1995).	Boosting a weak learning algorithm by majority.	Information and Computation,	121 (2), 256-285	

Each individual segmented string  $x_i$  is converted to a pairwise training example by creating a coreferent training pair  $(x_i, x_i)$ , which allows accumulating expectations of emissions for characters or tokens in that string along with accumulating the expectations of cross-component transitions. Forward and backward procedures are modified for segmented string pairs so that expectations are only accumulated for component pair HMMs that produce alignments for the corresponding fields, while for unsegmented string pairs, expectations are accumulated over all component pair HMMs, thus considering alignments over all possible segmentations of the two strings.

Because the proposed model is designed for settings where supervision is limited, and the number of parameters in the above model can be very large, training may result in poor parameter estimates due to sparsity of training data. To address this, we employ a variant of shrinkage, or deleted interpolation – a smoothing technique previously used in generative models for language modeling (Jelinek & Mercer, 1980) and information extraction (Freitag & McCallum, 1999). We simultaneously train two models: one that emits actual observations (individual characters or tokens for character-based and token-based edit distances respectively), and another that distinguishes between several large classes of emissions (characters, digits and punctuation for character-based edit distance, and vocabulary, non-vocabulary, and numeric tokens for token-based edit distance). Parameters of the two models are then interpolated ("shrunk") using the method of Freitag and McCallum (1999).

#### **Experimental Results**

We perform evaluation following the cross-validation procedure described in Section 3.1.1 on two datasets where segmentation is available: *Restaurant* and *Cora. Restaurant* is a database obtained by Tejada et al. (2002), who integrated records from Fodor's and Zagat's guidebooks to obtain 864 restaurant names and addresses that include 112 duplicates. *Cora* is a collection of 1295 distinct citations to 122 Computer Science research papers from the Cora Computer Science research paper search engine collected by McCallum et al. (2000). The citations were automatically segmented into multiple fields such as *author, title, venue*,

Figure 3.10: Sample coreferent records from the Cora dataset

author	title	venue	year
W. W. Cohen, R. E. Shapire,	Learning to order things.	In Advances in Neural Informa-	1998
and Y. Singer.		tion Processing Systems 10,	
William W. Cohen, Rob	Learning to order things.	To appear in NIPS-97,	1997
Schapire, and Yoram Singer.			

Figure 3.11: Sample coreferent records from the *Restaurant* dataset

name	address	city	phone	cuisine
Fenix	8358 Sunset Blvd. West	Hollywood	213/848-6677	American
Fenix at the Argyle	8358 Sunset Blvd.	W. Hollywood	213-848-6677	French(new)

etc. by an information extraction system, resulting in some noise in the field values. Figures 3.10 and 3.11 present sample coreferent records from the two datasets in segmented form.

Since SPHMM is designed for domains where entities are represented by strings containing multiple fields, we omit the available segmentation for all records in the test fold, while retaining it in the training fold for segmented supervision of types of (i) and (iii) in the list above. For *Cora*, five fields are distinguished: *author, title, venue, year,* and *other*, where the *other* field may includes such information as page numbers, names of editors, location, etc. For *Restaurant*, fields *name, street address, city,* and *cuisine* are distinguished. We employ token-based edit distance in all experiments, since in these domains the differences between the fields are mainly at the token, not character level.

We compare the accuracy of spHMM-learned affine-gap edit distance with the following baselines:

- PHMM: learnable affine-gap edit distance without segmentation described in Section 3.1.1;
- SEQ: a baseline that uses labeled and unlabeled strings to train the IE system of Grenager, Klein, and Manning [2005] that was specifically designed to handle unsupervised data. Individual affine-gap edit distances are learned for all extracted fields, and an SVM classifier is trained to combine them; Section 3.2 describes this process in

detail. During testing, the IE system segments each string into fields. Learned affinegap edit distances are computed for all extracted fields, and then combined using the SVM classifier to obtain overall string similarity estimates.

Comparison with the PHMM baseline evaluates whether incorporating segmentation in learnable affine-gap edit distance yields improvements, while comparison with the SEQ baseline evaluates performing the segmentation and string matching steps sequentially.

We consider four combinations of training data for the spHMM: segmented string pairs only, a mixture of segmented and unsegmented pairs, a mixture of unsegmented and individual segmented strings, and unsegmented pairs only. In all experiments 50 string pairs are used; the three numbers in identifiers SPHMM-50-0-0, SPHMM-25-25-0, SPHMM-0-50-50, and SPHMM-0-50-0 represent the amount of supervision for the three supervision types in the order listed in the previous section. For example, SPHMM-25-25-0 uses 25 segmented coreferent pairs, 25 unsegmented coreferent pairs, and no individual segmented strings for training. The SPHMM-50-0-0, SPHMM-25-25-0, and SPHMM-0-50-0 curves demonstrate the effects of training on various combinations of segmented and unsegmented pairwise supervision, while the SPHMM-0-50-50 curve shows the effects of adding some individual segmented supervision to the pairwise unsegmented supervision.

Figures 3.12 and 3.13 contain precision-recall curves for the two datasets. The results demonstrate that affine-gap edit distance based on spHMM outperforms both regular learnable affine-gap edit distance as well as the sequential combination of segmentation and learnable affine-gap edit distance on both datasets. The improvement is less pronounced on the *Cora* dataset compared to the *Restaurant* dataset: this is due to the fact that the field structure in citations is more complex than in restaurant records, since the ordering of the fields varies significantly. As a result, learning an accurate segmentation model is more difficult for *Cora*. If field extraction is performed in isolation, segmentation errors degrade the quality of similarity computations significantly as can be seen from the SEQ results.



Figure 3.12: Field-level linkage results for the unsegmented Restaurant dataset



Figure 3.13: Field-level linkage results for the unsegmented Cora dataset

In contrast, SPHMM is able to improve over non-segmented learnable edit distance by combining similarities from the multiple alignments.

The utility of training the model on segmented versus unsegmented string pairs is also dependent on the difficulty of the segmentation task. Because segmentations produced by the trained model are less reliable in *Cora* than in *Restaurant*, utilizing more segmented training data does not result in statistically significant improvements. In *Restaurant* records, the field structure is more regular, and a small amount of either segmented pairs or individual segmented strings improves results obtained with just unsegmented pairs, as the differences between the SPHMM-0-50-0 and the other SPHMM results demonstrate.

Overall, the results show that incorporating segmentation into learnable edit distance yields an improved similarity function for string linkage even without segmented training data, while increased improvements are obtained when small amounts of segmented supervision is provided.

## 3.2 Learnable Record Similarity

#### 3.2.1 Combining Similarity Across Fields

Because correspondence between overall record similarity and individual field similarities can vary greatly depending on field importance, an accurate record similarity function must weigh fields in proportion to their contribution to the true similarity between records. For example, similarities of the *author* and *title* fields in bibliographic citation are more significant than similarity for the *year* field, and accurate distance measure for overall citations must reflect this. While statistical aspects of combining similarity scores for individual fields have been addressed in previous work on record linkage (Winkler, 1999), availability of labeled duplicates allows a more direct approach that uses a binary classifier which computes a similarity function (Tejada et al., 2002; Elfeky et al., 2002; Sarawagi & Bhamidipaty, 2002; Cohen & Richman, 2002). Given a database containing records composed of *k* 

	Name	Address	City	Cuisine
	Fenix	8358 Sunset Blvd. West	Hollywood	American
	Fenix at the Argyle	8358 Sunset Blvd.	W. Hollywood	French (new)
Field simila functio	rity $d_{1n}$ $d_{2n}$ ons Feature vector $d_1$	$d_{1a}  d_{2a}  d_{1c}$	$\begin{array}{c} d_{2c}  d_{1cu} \\ d_{2cu} \end{array}$	и <i>d<sub>2cu</sub></i> Т
			Ι	Distance
	(	Coreferent pairs Non	-coreferent pairs	

Figure 3.14: Computation of record similarity from individual field similarities

different fields and a set of *m* similarity functions,  $\{d_1(\cdot, \cdot), \ldots, d_m(\cdot, \cdot)\}$ , we can represent any pair of records by an *mk*-dimensional vector. Each component of the vector contains similarity between two field values computed using one of the *m* similarity functions.

As in training string similarity functions, pairs of coreferent records can be used to construct a training set of such feature vectors by assigning them a positive class label. Pairs of non-coreferent records form a complementary set of negative examples, which can be very large due to the paiwise nature of the matching task, and therefore requires subsampling; this problem is addressed in more detail in Section 3.3.

A binary classifier is then trained on such supervision to discriminate between pairs of records corresponding to coreferent and non-coreferent pairs. Previous work in this area relied on Bayesian classifiers (Winkler, 2002), decision trees (Tejada et al., 2002; Elfeky et al., 2002), and logistic regression (Cohen & Richman, 2002). We employ a Support Vector Machine (SVM) with an RBF kernel which in the last decade has proven to be a top-performing classifier on a wide array of categorization tasks (Shawe-Taylor & Cristianini, 2000). Properties that make SVMs particularly appropriate for discriminating between coreferent and non-coreferent record pairs include their resilience to noise, ability to handle correlated features, and robustness to the relative sizes of training samples from different classes. The latter requirement is particularly important, given that the proportion of coreferent records in a database is very difficult to estimate in realistic record linkage applications due to the pairwise nature of the task.

Once trained, the SVM provides a confidence estimate for each record pair which can be treated as an estimate of similarity between records. The confidence estimate is derived from the *margin* of a particular example, that is, its distance from the hyperplane that separates the two classes. It has been shown that margins can be converted to confidence or probability values via a logistic transformation (Wahba, 1999; Platt, 1999b).

Figure 3.14 illustrates the process of computing record similarity using multiple similarity measures over each field and an SVM to categorize the resulting feature vector as belonging to the class of duplicates or non-duplicates. For each field of the database, two similarity functions,  $d_1$  and  $d_2$ , are applied to compute similarity for that field. The values computed by these measures form the feature vector that is then classified by a support vector machine, producing a confidence value that represents similarity between the database records.

#### The Overall Record Linkage Framework

An overall view of our system, MARLIN (Multiply Adaptive Record Linkage with INduction), is presented in Figure 3.15. The training phase consists of two steps. First, the learnable string similarity functions are trained for each record field. The training corpus of field-level coreferent and non-coreferent pairs is obtained by taking pairs of values for each field from the set of coreferent record pairs. Because equivalent records may contain individual fields that are not coreferent, training data can be noisy. For example, if one record describing a restaurant contains "*Asian*" in the *cuisine* field, and an equivalent record contains "*Seafood*", a noisy training pair is formed that implies equivalence between these two strings. However, this issue does not pose a serious problem for our approach for two reasons. First, particularly noisy fields that are unhelpful for identifying record-level duplicates will be considered irrelevant by the classifier that combines similarities from different

#### Training:



Figure 3.15: MARLIN overview

fields. Second, the presence of such pairs in the database indicates that there is a degree of similarity between such values, and using them in training allows the learnable record similarity function to capture that likelihood as much as possible.

After individual string similarity functions are learned, they are used to compute distances for each field of training record pairs to obtain training data for the binary classifier in the form of vectors composed of distance features.

The record linkage phase starts with generation of potential coreferent pairs. Since producing all possible pairs of records and computing similarity between them is too expensive for large databases, MARLIN incorporates several blocking strategies to efficiently obtain candidate record pairs that are approximately similar and warrant detailed distance computations. Blocking is discussed in detail in Chapter 5, in which we describe an adaptive framework for training blocking functions.

Learned string similarity functions are then used to calculate distances for each field of every candidate record pair, forming field feature vectors for the classifier. Confidence estimates for belonging to the class of coreferent pairs are produced by the binary classifier for each candidate pair, and pairs are sorted by decreasing similarity to evaluate similarity function accuracy as discussed in Section 3.1.1.

#### **3.2.2 Experimental Results**

We evaluated the performance of multi-field record linkage within the MARLIN framework using the SVM implementation within the WEKA software toolkit (Witten & Frank, 1999) that relies on the Sequential Minimal Optimization (SMO) training algorithm for the underlying quadratic optimization problem (Platt, 1999a). We conducted two sets of experiments. First, we compared the performance of learnable and non-learnable variants of affine-gap edit distance as components of a record-level similarity function that combines their predictions for individual fields. We have again used the *Restaurant* and *Cora* datasets, this time using the field segmentation provided with the datasets.

Figures 3.16 and 3.17 present the precision-recall curves for record linkage using SVM as the combining classifier and different field-level similarity functions: learned edit distance, unlearned edit distance, and TF-IDF weighted cosine similarity. The results demonstrate that using learnable string edit distance with affine gaps leads to improved performance even when similarities from multiple fields are combined. At high recall levels (above 90%), using learnable edit distance performs particularly well, indicating that it provides better field similarity estimates for particularly difficult coreferent pairs, leading to more accurate computation of the overall record similarity.

Second, we compared the performance of several classifiers that have been recently employed for the record linkage task by different researchers. Using the implementations in the WEKA toolkiet, we compared the following classifiers using both unlearned and



Figure 3.16: Record-level linkage results on the Cora dataset



Figure 3.17: Record-level linkage results on the Restaurant dataset

learnable affine-gap edit distances as the underlying field similarity functions:

- SVM-RBF: Support Vector Machine with the Gaussian kernel;
- SVM-LINEAR: Support Vector Machine with the linear kernel;
- ADABOOST: boosting algorithm of Freund and Schapire (1996) that uses J48, WEKA's implementation of C4.5 decision tree as the base classifier (Quinlan, 1993);
- MAXENT: logistic regression (le Cessie & van Houwelingen, 1992);
- BAYESNET: a Bayesian Network learner that uses the K2 structure learning algorithm (Cooper & Herskovits, 1992).

Figures 3.19 and 3.20 present results for the experiments that used learnable affinegap edit distance on *Restaurant* and *Cora* datasets, while mean average precision (MAP) values for all experiments are shown in Figure 3.18.

Overall, the results demonstrate that Support Vector Machines yield the best accuracy on both datasets, outperforming the other classifiers significantly. Both the Gaussian and the linear kernel provide equivalently good performance, which is not surprising since the classification is performed on a very low-dimensional problem. Other classifiers perform significantly worse for both datasets. We conclude that SVM-based learnable record similarity is a robust, accurate similarity function for combining similarities of multiple fields in the record linkage setting. We also note that using learnable affine-gap edit distance as the field similarity function provides better results than using unlearned edit distance, although statistically significant differences are only observed on parts of the learning curve for most classifiers (e.g., for SVM-RBF and SVM-LINEAR the improvements are statistically significant at 0.05 level using a two-tailed paired t-test only at 98% and 100% recall respectively). However, the improvements are consistent and suggest that using learnable edit distance for field-level comparisons leads to accuracy improvements even when fields are combined by a classifier.

	Rest	taurant	Cora		
	pHMM ED	pHMM ED Unlearned ED		Unlearned ED	
SVM-RBF	0.999	0.996	0.998	0.997	
SVM-LINEAR	0.994	0.994	0.998	0.997	
AdaBoost	0.948	0.927	0.975	0.974	
MaxEnt	0.938	0.937	0.824	0.815	
BAYESNET	0.884	0.873	0.976	0.967	

Figure 3.18: Mean average precision values for record-level linkage

# 3.3 Training-Set Construction for Learning Similarity Functions

Training string and record similarity functions in real-world scenarios requires selecting a set of pairs for a human expert to label as coreferent or non-coreferent, or asking the expert to identify all groups of coreferent records in the dataset, which is not feasible for large datasets. Since typical corpora and databases contain few coreferent records, selecting random pairs as potential training examples leads to training sets with extremely few coreferent pairs (positive examples). As a result, such randomly selected training sets are highly skewed toward non-coreferent pairs, which leads to suboptimal performance of similarity functions trained on this data. We propose two heuristic approaches for collecting training data: static-active learning and weakly-labeled selection, and present experimental results on their effectiveness.

#### 3.3.1 Likely-positive Selection of Training Pairs

Traditional active learning systems are "dynamic": labels of training examples selected in earlier rounds influence which unlabeled examples are deemed most informative in subsequent rounds. While prior work has examined dynamic active learning approaches to adaptive record linkage (Sarawagi & Bhamidipaty, 2002; Tejada et al., 2002), such strategies may not always be feasible due to high computational costs exacerbated by the large number of potential training examples. We propose using a "static" active learning method for selecting pairs of records that are *likely* to be coreferent, as a middle ground between



Figure 3.19: Classifier comparison for record-level linkage on the Cora dataset



Figure 3.20: Classifier comparison for record-level linkage on the Restaurant dataset

computationally expensive dynamic active learning methods that try to identify the most informative training examples and random selection that is efficient but fails to select useful training data.

Our approach relies on the fact that off-the-shelf string similarity functions, such as TF-IDF cosine similarity, can accurately identify coreferent strings or records at low recall levels (high confidence) even when coreferent and non-coreferent pairs are difficult to distinguish at high recall levels (low confidence). Therefore, when a random sample of records from a database is taken and similarity between them is computed using such an off-the-shelf similarity function, string or record pairs with high similarity scores are likely to be coreferent. By asking the user to label strings or records with high textual similarity, a training sample with a high proportion of coreferent pairs can be obtained. At the same time, non-coreferent pairs selected using this method are likely to be "near-miss" negative examples that are more informative for training than randomly selected record pairs most of which tend to be "obvious" non-coreferent pairs. Because training sets constructed using this method have a dramatically different distribution of coreferent and non-coreferent pairs from their actual distribution in the dataset, adding some randomly selected non-coreferent pairs is desirable to decrease the difference between the two distributions and provide the learner more negative examples.

Figures 3.21 and 3.22 demonstrate the comparative utility of static-active selection and random selection for choosing training record pairs on *Restaurant* and *Cora* datasets respectively. The record similarity function was trained on 40 training examples comprised of randomly selected record pairs and/or the most similar pairs selected by a static-active method using TF-IDF cosine similarity. Using a token-based inverted index for the vectorspace model (Baeza-Yates & Ribeiro-Neto, 1999) allowed efficient selection of static-active training examples without computing similarity between all pairs of records. All experiments utilized SVM<sup>*light*</sup> for computing learnable record similarity function and two unlearned string similarity functions for field comparisons: TF-IDF cosine similarity and edit



Figure 3.21: Comparison of random and likely-positive training example selection on the *Restaurant* dataset



Figure 3.22: Comparison of random and likely-positive training example selection on the *Cora* dataset

distance with affine gaps.

For both datasets, the highest performance is achieved when record similarity functions are trained using a mix of static-active and randomly selected pairs. However, employing many random pairs with a few static-active examples yields the best results on *Cora*, while on *Restaurant* the highest performance is achieved when the system is trained on a balanced mix of static-active and random examples. This difference is explained by the makeup of the two datasets. Cora has a higher absolute number of coreferent pairs than Restaurant (8,592 versus 56 for each fold); coreferent pairs in Cora also represent a larger proportion of all record pairs (4.1% versus 0.06% for each fold). On Restaurant, random selection results in training sets that contain almost no coreferent pairs, while including a significant number of pairs selected using the static-active technique leads to balanced training sets that contain sufficient positive and negative examples. On Cora, however, randomly selected record pairs are likely to contain a few coreferent pairs. Including a limited number of record pairs chosen using the static-active technique results in the best performance, but as more static-active examples are added, performance decreases because highly similar coreferent pairs take the place of informative non-coreferent pairs in the training set. Thus, the worst performance on *Restaurant* occurs when all training examples are chosen randomly because coreferent pairs are almost never encountered, while on *Cora* using only examples chosen by static-active selection results in the opposite problem: extremely few non-coreferent pairs are found, and the class distribution of training data is highly skewed toward non-coreferent pairs.

Based on these results, we conclude that best training sets for learnable record similarity functions are obtained when randomly chosen pairs of records are combined with pairs chosen using static-active selection. The specific proportion in which the two kinds of training data should be mixed can be estimated based on the outcome of labeling randomly chosen pairs. If coreferent pairs are exceptionally rare, a significant number of static-active examples is required to obtain a sufficient sample of coreferent pairs, while databases with a large number of coreferent records need only a small number of record pairs selected using the static-active methodology to complete a representative training set.

Overall, we show that a reasonable baseline to which dynamic active learning methods for adaptive similarity functions should be compared is not the one that uses only randomly selected training pairs, but one that employs the static-active method to overcome the extreme skewness in class distribution that is typical for similarity function learning and record linkage problems.

#### **3.3.2 Weakly-labeled Selection**

While the static-active method allows identifying coreferent training pairs for learnable similarity functions, the inverse problem can be encountered in some real-world situations: a "legacy" training set consisting of identified coreferent pairs may be available, while informative non-coreferent pairs need to be collected. For such situations we consider an unsupervised technique for obtaining negative examples. Since coreferent records are rare in a typical database, two randomly selected records are likely to be non-coreferent, and therefore can potentially be used as negative training examples for learning similarity functions. To help ensure that no coreferent records are included among these pairs, only pairs of records that do *not* share a significant number of common tokens should be included as negative examples. Such selection of "weakly-labeled" (and potentially noisy) non-coreferent record pairs is the unsupervised analog of static-active selection of coreferent pairs. The process can also be thought of as the opposite of blocking or canopies techniques that use off-the-shelf metrics to avoid comparing "obvious" non-coreferent records to speed up the record linkage process.

We compared the record linkage accuracy of MARLIN trained on weakly-labeled negatives with training on user-labeled negatives. Figures 3.23 and 3.24 present the results of these experiments on the *Restaurant* and *Cora* datasets. Weakly-labeled negatives were selected randomly from record pairs that shared no more than 20% of tokens to minimize the



Figure 3.23: Comparison of using weakly-labeled non-coreferent pairs with using random labeled record pairs on the *Restaurant* dataset



Figure 3.24: Comparison of using weakly-labeled non-coreferent pairs with using random labeled record pairs on the *Cora* dataset

noise. All experiments used training sets composed of two parts: half the examples were positives randomly selected among user-labeled coreferent pairs, and the other half was composed of either weakly-labeled non-coreferent records or randomly selected labeled record pairs. SVM<sup>*light*</sup> was employed to compute record similarity, and TF-IDF cosine similarity and edit distance with affine gaps were used as the underlying string similarity functions for individual fields.

The results again demonstrate that the utility of the heuristic selection of training data for similarity function learning is dataset-dependent. On *Restaurant*, where coreferent pairs are scarce and randomly selected records are truely non-coreferent with very high probability, using weakly-labeled non-coreferent pairs yields results identical to randomly selected labeled coreferent pairs when a large number of examples is selected, and improves slightly over random selection when the training set is small. We conjecture that biasing the SVM with "negative but slightly similar" examples when very little training data is available allows learning a better separating hyperplane. On *Cora*, using weakly-labeled negatives leads to slight degradation of system accuracy, which is expected since coreferent pairs are relatively frequent, and noise is likely to be introduced when negative examples are collected in an unsupervised manner. However, the drop in performance is small, and in situations where human labeling of negatives is expensive or infeasible (e.g. due to privacy issues), using weakly-labeled selection is a viable avenue for unsupervised acquisition of negative training.

### **3.4 Related Work**

Several researchers described methods for learning string similarity functions in prior work. For string edit distance, Ristad and Yianilos (1998) proposed learning the costs of individual edit operations of Levenshtein distance using a probabilistic generative model. In their model, a string alignment is equivalent to a sequence of character pairs generated by edit operations emitted by a hidden Markov model with a single non-terminal state. We have followed the same approach in developing a learning model for affine-gap edit distance, which provides significantly better similarity estimates for natural text strings (Bilenko & Mooney, 2002; Cohen et al., 2003a).

Both our model and the model of Ristad-Yianilos are instances of pair Hidden Markov Models, proposed earlier for biological sequence alignments in bioinformatics (Durbin et al., 1998). Using such models for record linkage requires several modifications that we have described. Among those, parameter tying, gap-to-gap transitions, and length normalization are important for obtaining good performance of pair HMM-based edit distance in natural language string similarity computations.

Two other models for learning the costs of individual edit operations have been proposed by Zhu and Ungar (2000) and Yancey (2004). Zhu and Ungar (2000) have used genetic algorithms for learning the costs of several manually constructed edit operations. Yancey (2004) has employed a variant of Expectation-Maximization for learning the probabilities of individual edit operations, where only highest-probability (Viterbi) alignments were used to accumulate expectations. Both of these approaches are adaptive variants of Levenshtein distance and do not include taking gaps into account.

In recent years, two models of learnable edit distance have been proposed based on discriminative classifiers. Joachims (2003) formulated the problem of learning edit operation costs as maximum-margin optimization, and showed how it can be solved using SVMs. However, this formulation relies on availability of actual string alignments, not just coreferent string pairs, and therefore requires significant labeling effort to obtain training data. McCallum, Bellare, and Pereira (2005) described a model for learning the parameters of affine-gap edit distance based on Conditional Random Fields (CRFs), a discriminative analog of HMMs. While they have obtained improvements over the field-level results we presented in Section 3.1.1 on some of the datasets, their method relies on a number of extra matching features, some of which could also be implemented in the HMM-based model. Additionally, training algorithms for CRF-based models are more complex than EM-based training of HMMs and incur significant computational costs.

A number of record linkage researchers have relied on classifiers to combine similarity estimates across multiple fields. Approaches in the statistical literature has traditionally relied on generative classifiers such as Naive Bayes and Bayesian networks (Winkler, 2002), while in recent machine learning research a number of classifiers have been used, including decision trees (Elfeky et al., 2002; Tejada et al., 2002; Sarawagi & Bhamidipaty, 2002) and logistic regression (Cohen & Richman, 2002). We have shown that Support Vector Machines outperform these methods significantly on both datasets that we considered.

Sarawagi and Bhamidipaty (2002) and Tejada et al. (2002) have proposed active learning methods that obtain informative training examples for learning record-level similarity functions between records. The training set construction strategies we described in Section 3.3 approximate these methods without the computational cost of active learning for selecting likely positives, and without the need for a human oracle for weak negatives.

Recent work on record linkage has focused on the third stage of the record linkage process described in Section 2.2: clustering for obtaining groups of coreferent records. In particular, a number of methods have been proposed for *collective* grouping coreferent records and obtaining the complete partitioning of datasets into such groups (Pasula et al., 2003; Wellner et al., 2004; Li et al., 2004; Singla & Domingos, 2005). Our work addresses an orthogonal problem, accurate computation of record and field similarities, and the methods presented in this chapter can be used as input to the collective linkage approaches, since they rely on pairwise similarity estimates between records or their fields.

## **3.5 Chapter Summary**

In this chapter, we have shown how learnable similarity functions lead to significant performance improvements in the record linkage task. Because record linkage requires accurate distance estimates between individual field values and overall records, adapting similarity functions that provide these estimates allows learning domain-specific parameters to compute similarity with higher accuracy.

Two learnable variants of affine-gap edit distance based on pair HMMs that we described learn edit operation and gap costs that discriminate between coreferent and noncoreferent strings. For record-level similarity, we have shown that using Support Vector Machines leads to accurate distance estimations between records composed of multiple fields. We have demonstrated that employing learnable field-level similarity functions is still advantageous over using unlearned methods in multi-field domains when field similarities are combined by a classifier. Finally, we have shown that informative training examples for these methods can be collected without relying on active learning methods, and possibly without even relying on human supervision.

# **Chapter 4**

# Learnable Similarity Functions in Semi-supervised Clustering

In this chapter, we show how learnable similarity functions improve clustering accuracy when employed in a semi-supervised clustering setting. We describe a probabilistic model for semi-supervised clustering based on Hidden Markov Random Fields (HMRFs) that accommodates a wide variety of learnable similarity functions. This model yields a clustering algorithm, HMRF-KMEANS, that integrates similarity function learning with constraint-based clustering, improving on algorithms that perform these tasks in isolation.

### 4.1 Similarity Functions in Clustering

As discussed in Section 2.3, clustering inherently relies on similarity estimations as its goal is to group instances that are alike while separating instances that are dissimilar. For many datasets, off-the-shelf functions may fail in providing similarity estimates that place same-cluster points nearby and different-cluster points far apart, preventing the discovery of a desired partitioning of a dataset. Examples of same-cluster and different-cluster instance pairs that are available in the semi-supervised clustering setting provide supervision for
training the similarity function to produce appropriate distance estimates, making it easier to create clusters that respect the pairwise supervision when grouping the unlabeled data.

For some datasets, clusters of different shapes may be desirable, which effectively indicates that datapoints in these clusters lie in different subspaces of the overall data space. Recovering such partitioning requires using an individual similarity function for each cluster, a fact that is exploited in unsupervised clustering algorithms like Expectation-Maximization that estimate distinct density parameters for different clusters. In the semisupervised setting, pairwise constraints provide additional information about the shape of underlying clusters that can be captured if the similarity function is learned using both supervised and unsupervised data.

The HMRF framework for semi-supervised clustering presented below addresses the above considerations in a principled probabilistic model and leads to a clustering algorithm, HMRF-KMEANS, that combines the advantages of constraint-based and similaritybased approaches to semi-supervised clustering. The following section presents an overview of the overall HMRF framework, more detailed description of which can be found in (Basu et al., 2006). Then, use of learnable similarity functions within the framework is described in detail. Three examples of similarity functions and their parameterizations for use with HMRF-KMEANS are provided for squared Euclidean distance, cosine distance and KL divergence. Through parameterization, each of these functions becomes adaptive in the semi-supervised clustering setting, which allows learning the appropriate notion of similarity using both the pairwise constraints and the unlabeled data.

# 4.2 The HMRF Model for Semi-supervised Clustering

We assume that we are given a set of *n* data points  $\mathcal{X} = \{x_i\}_{i=1}^n$ , where each  $x_i \in \mathbb{R}^d$  is a *d*-dimensional vector. Supervision consists of two sets of pairwise constraints over points in  $\mathcal{X}$ : must-link constraints  $C_{ML} = \{(x_i, x_j)\}$  and cannot-link constraints  $C_{CL} = \{(x_i, x_j)\}$ , where  $(x_i, x_j) \in C_{ML}$  implies that  $x_i$  and  $x_j$  should belong to the same cluster, while  $(x_i, x_j) \in C_{ML}$ 

 $C_{CL}$  implies that  $x_i$  and  $x_j$  should belong to different clusters. The constraints may be accompanied by associated violation costs W, where  $w_{ij}$  represents the cost of violating the constraint between points  $x_i$  and  $x_j$ , if such a constraint exists.

The model relies on selecting a *distortion measure*  $d_{\mathcal{A}}$  to compute dissimilarity between points:  $d_{\mathcal{A}} : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ . The distortion measure corresponds to a learnable similarity function, with  $\mathcal{A}$  being the set of parameters to learn, which is typically a matrix or a vector of weights. The objective of semi-supervised clustering is to partition the datapoints  $\mathcal{X}$  into K disjoint clusters  $\{\mathcal{X}_1, \ldots, \mathcal{X}_K\}$  so that the total distortion between the points and the corresponding cluster representatives is minimized according to the given distortion measure  $d_{\mathcal{A}}$ , while constraint violations are kept to a minimum.

### 4.2.1 HMRF Model Components

The Hidden Markov Random Field (HMRF) (Zhang, Brady, & Smith, 2001) probabilistic framework for semi-supervised constrained clustering consists of the following components:

- An observable set X = {x<sub>i</sub>}<sup>n</sup><sub>i=1</sub> corresponding to the given data points X. Note that we overload notation and use X to refer to both the given set of data points and their corresponding random variables.
- An *unobservable* (hidden) set \$\mathcal{Y} = {y\_i}\_{i=1}^n\$ corresponding to cluster assignments of points in \$\mathcal{X}\$. Each hidden variable \$y\_i\$ encodes the cluster label of the point \$x\_i\$ and takes values from the set of cluster indices {1,...,K}.
- An *unobservable* (hidden) set of generative model parameters Θ, which consists of distortion measure parameters A and cluster representatives M = {μ<sub>i</sub>}<sup>K</sup><sub>i=1</sub>: Θ = {A, M}.
- An *observable* set of constraint variables  $C = \{c_{12}, c_{13}, \dots, c_{n-1,n}\}$ . Each  $c_{ij}$  is a tertiary variable taking on a value from the set  $\{-1, 0, 1\}$ , where  $c_{ij} = 1$  indicates



Figure 4.1: A Hidden Markov Random Field for semi-supervised clustering

that  $(x_i, x_j) \in C_{ML}$ ,  $c_{ij} = -1$  indicates that  $(x_i, x_j) \in C_{CL}$ , and  $c_{ij} = 0$  corresponds to pairs  $(x_i, x_j)$  that are not constrained.

Fig. 4.1 shows the HMRF for a hypothetical five-point dataset X. The datapoints correspond to variables  $(x_1, \ldots, x_5)$  that have cluster labels  $\mathcal{Y} = (y_1, \ldots, y_5)$ , which may each take on values (1, 2, 3) denoting the three clusters. Three pairwise constraints are provided: two must-link constraints  $(x_1, x_2)$  and  $(x_1, x_4)$ , and one cannot-link constraint  $(x_2, x_3)$ . Corresponding constraint variables are  $c_{12} = 1$ ,  $c_{14} = 1$ , and  $c_{23} = -1$ ; all other variables in *C* are set to zero. The task is to partition the five points into three clusters. Fig. 4.1 demonstrates one possible clustering configuration which does not violate any constraints. The must-linked points  $x_1, x_2$  and  $x_4$  belong to cluster 1; the point  $x_3$ , which is cannot-linked with  $x_2$ , is assigned to cluster 2;  $x_5$ , which is not involved in any constraints, belongs to cluster 3.



Figure 4.2: Graphical plate model of variable dependence in HMRF-based semi-supervised clustering

# 4.2.2 Joint Probability in the HMRF Model

The graphical plate model (Buntine, 1994) of the dependence between the random variables in the HMRF is shown in Figure 4.2, where the unshaded nodes represent the hidden variables, the shaded nodes are the observed variables, the directed links show dependencies between the variables, while the lack of an edge between two variables implies conditional independence. The prior probability of  $\Theta$  is assumed to be independent of *C*. The probability of observing the label configuration  $\mathcal{Y}$  depends on the constraints *C* and current generative model parameters  $\Theta$ . Observed datapoints corresponding to variables *X* are generated using the model parameters  $\Theta$  based on cluster labels  $\mathcal{Y}$ , independent of the constraints *C*. The variables *X* are assumed to be mutually independent: each  $x_i$  is generated individually from a conditional probability distribution  $Pr(x_i|y_i, \Theta)$ . Then, the joint probability of *X*,  $\mathcal{Y}$ , and  $\Theta$ , given *C*, can be factorized as follows:

$$\Pr(\mathcal{X}, \mathcal{Y}, \Theta | \mathcal{C}) = \Pr(\Theta) \ \Pr(\mathcal{Y} | \Theta, \mathcal{C}) \ \prod_{i=1}^{n} p(x_i | y_i, \Theta)$$
(4.1)

where  $p(\cdot|y_i, \Theta)$  is the parameterized probability density function for the  $y_i$ -th cluster, from which  $x_i$  is generated. This probability density corresponds to the clustering distortion measure  $d_{\mathcal{A}}$ , and is discussed in detail in Section 4.3 below.

Each hidden random variable  $y_i \in \mathcal{Y}$  representing the cluster label of  $x_i \in X$  is associated with a set of neighbors  $\mathcal{N}_i$ , defined as all points to which  $x_i$  is must-linked or cannot-linked:  $\mathcal{N}_i = \{y_j | (x_i, x_j) \in C_{ML} \cup (x_i, x_j) \in C_{CL}\}$ . We make the Markov assumption that each label  $y_i$  is conditionally independent of all other labels in  $\mathcal{Y}$  given the labels of its neighbors. The resulting random field over the hidden variables  $\mathcal{Y}$  is a Markov Random Field (MRF), in which by the Hammersley-Clifford theorem (Hammersley & Clifford, 1971) the prior probability of a particular label configuration  $\mathcal{Y}$  can be expressed as a Gibbs distribution (Geman & Geman, 1984):

$$\Pr(\mathcal{Y}|\Theta, \mathcal{C}) = \frac{1}{Z} \exp\left(-\sum_{i,j} \nu(i,j)\right)$$
(4.2)

where *Z* is the partition function (normalizing term), and each v(i, j) is the potential function encoding the compatibility of cluster labels  $y_i$  and  $y_j$ . Because label compatibility is only relevant for pairs of points that participate in constraints, we define v(i, j) as follows:

$$v(i,j) = \begin{cases} w_{ij} f_{ML}(i,j) & \text{if } c_{ij} = 1 \text{ and } y_i \neq y_j \\ w_{ij} f_{CL}(i,j) & \text{if } c_{ij} = -1 \text{ and } y_i = y_j \\ 0 & \text{otherwise} \end{cases}$$
(4.3)

where  $f_{ML}$  and  $f_{CL}$  are *penalty functions* that encode the lowered probability of observing configurations of  $\mathcal{Y}$  where must-link and cannot-link constraints are violated respectively, and  $w_{ij}$  is the user-provided constraint weight that can be used to indicate its importance. Penalty functions are chosen to correlate with the distortion measure by depending on the distortion measure parameters  $\mathcal{A}$ , and will be described in detail in Section 4.3 below. Overall, this formulation for observing the label assignment (clustering)  $\mathcal{Y}$  results in higher probabilities being assigned to configurations in which cluster assignments do not violate the provided constraints. Then, joint probability on the HMRF can be expressed as follows:

$$\Pr(\mathcal{X}, \mathcal{Y}, \Theta | \mathcal{C}) = \Pr(\Theta) \left( \frac{1}{Z} \exp\left( -\sum_{(i,j): c_{ij} \neq 0} v(i,j) \right) \right) \left( \prod_{i=1}^{n} p(x_i | y_i, \Theta) \right)$$
(4.4)

The first factor in the above expression describes a probability distribution over the model parameters preventing them from attaining degenerate values, thereby providing regularization. The second factor is the conditional probability of observing a particular label configuration given the provided constraints, effectively assigning a higher probability to configurations where the cluster assignments do not violate the constraints. Finally, the third factor is the conditional probability of generating the observed data points given the labels and the parameters: if maximum-likelihood estimation (MLE) was performed on the HMRF, the goal would have been to maximize this term in isolation.

Overall, maximizing the joint HMRF probability in Eq.(4.4) is equivalent to jointly maximizing the likelihood of generating datapoints from the model and the probability of label assignments that respect the constraints, while regularizing the model parameters.

# 4.3 Learnable Similarity Functions in the HMRF Model

The Joint probability formulation in Eq.(4.4) provides a general framework for incorporating various similarity functions in clustering by choosing a particular form of  $p(x_i|y_i, \Theta)$ , the probability density that generates the *i*-th instance  $x_i$  from cluster  $y_i$ . In this work, we restrict our attention to probability densities from the exponential family, where the expectation parameter corresponding to cluster  $y_i$  is  $\mu_{y_i}$ , the mean of the points of that cluster. Using this assumption and the bijection between regular exponential distributions and regular Bregman divergences (Banerjee et al., 2005b), the conditional density for observed data can be represented as follows:

$$p(x_i|y_i,\Theta) = \frac{1}{Z_{\Theta}} \exp\left(-d_{\mathcal{A}}(x_i,\mu_{y_i})\right),\tag{4.5}$$

where  $d_{\mathcal{R}}(x_i, \mu_{y_i})$  is the Bregman divergence between  $x_i$  and  $\mu_{y_i}$ , corresponding to the exponential density p, and  $Z_{\Theta}$  is the normalizer. Different similarity functions can be expressed via this exponential form:

- If  $x_i$  and  $\mu_{y_i}$  are vectors in Euclidean space, and  $d_{\mathcal{A}}$  is the square of the  $L_2$  distance parameterized by a positive semidefinite weight matrix  $\mathcal{A}$ ,  $d_{\mathcal{A}}(x_i, \mu_{y_i}) = ||x_i - \mu_{y_i}||_{\mathcal{A}}^2$ , then the cluster conditional probability is a *d*-dimensional multivariate normal density with covariance matrix  $\mathcal{A}^{-1}$ :  $p(x_i|y_i, \Theta) = \frac{1}{(2\pi)^{d/2}|A|^{-1/2}} \exp(-\frac{1}{2}(||x_i - \mu_{y_i}||_{\mathcal{A}}^2)$  (Kearns, Mansour, & Ng, 1997);
- If x<sub>i</sub> and μ<sub>yi</sub> are probability distributions, and d<sub>A</sub> is KL-divergence (d<sub>A</sub>(x<sub>i</sub>, μ<sub>yi</sub>) = Σ<sup>d</sup><sub>m=1</sub> x<sub>im</sub> log x<sub>im</sub>/μ<sub>yim</sub>), then the cluster conditional probability is a multinomial distribution (Dhillon & Guan, 2003).

The relation in Eq.(4.5) holds even if  $d_{\mathcal{A}}$  is not a Bregman divergence but a directional distance measure such as cosine distance. Then, if  $x_i$  and  $\mu_{y_i}$  are vectors of unit length and  $d_{\mathcal{A}}$  is one minus the dot-product of the vectors  $\left(d_{\mathcal{A}}(x_i, \mu_{y_i}) = 1 - \frac{\sum_{m=1}^{d} x_{im} \mu_{y_im}}{\|x_i\| \|\mu_{y_i}\|}\right)$ , then the cluster conditional probability is a von-Mises Fisher (vMF) distribution with unit concentration parameter (Banerjee et al., 2005a), which is the spherical analog of a Gaussian.

Putting Eq.(4.5) into Eq.(4.4) and taking logarithms gives the following clustering objective function, minimizing which is equivalent to maximizing the joint probability over the HMRF in Eq.(4.4):

$$\mathcal{J}_{\text{obj}} = \sum_{x_i \in \mathcal{X}} d_{\mathcal{A}}(x_i, \mu_{y_i}) + \sum_{c_{ij} \in \mathcal{C}} v(i, j) - \log \Pr(\Theta) + \log Z + n \log Z_{\Theta}$$
(4.6)

Thus, an optimal clustering is obtained by minimizing  $\mathcal{J}_{obj}$  over the hidden variables  $\mathcal{Y}$  and parameters  $\Theta$ , which are comprised of cluster centroids  $\mathcal{M}$  and distortion measure parameters  $\mathcal{A}$  (note that given cluster assignments  $\mathcal{Y}$ , the means  $\mathcal{M} = {\{\mu_i\}}_{i=1}^K$  are uniquely determined).

Selecting an appropriate distortion measure  $d_{\mathcal{A}}$  for a clustering task typically involves knowledge about properties of the particular domain and dataset. For example, squared Euclidean distance is most appropriate for low-dimensional data, while cosine similarity is most fitting for data described by vectors in high-dimensional space where directional differences are important but vector lengths are irrelevant.

Once a distortion measure is chosen for a given domain, the functions  $f_{ML}$  and  $f_{CL}$  must be defined to penalize must-link and cannot-link constraint violations respectively, as described in Section 4.2.2. Each violation penalty is scaled proportionally to the "egregiousness" of the violation with respect to the current similarity function. That is, a violated must-link constraint carries a heavy penalty in the objective function if the distance between its points is high: this indicates that the two points are highly dissimilar, and the current parameterization of the similarity function is grossly inadequate. Likewise, two if points of a violated cannot-link constraints are similar, the penalty is high since the parameterization of the similarity function is inappropriate: the points should be dissimilar.

To reflect this intuition, the penalty functions are defined as follows:

$$f_{ML}(i,j) = \boldsymbol{\varphi}(i,j) \tag{4.7}$$

$$f_{CL}(i,j) = \varphi^{\max} - \varphi(i,j)$$
(4.8)

where  $\varphi: X \times X \to \mathbb{R}^+$  is a non-negative function that penalizes constraint violations, while  $\varphi^{\max}$  is an upper bound on the maximum value of  $\varphi$  over any pair of points in the dataset; examples of such bounds for specific distortion functions are described below. The function  $\varphi$  is chosen to be identical or proportional to the distortion measure, assigning higher penalties to violations of must-link constraints between points that are distant with respect to the current parameter values of the distortion measure. Conversely, penalties for violated cannot-link constraints are higher for points that have low distance between them. With this formulation of the penalty functions, constraint violations will lead to changes in the

distortion measure parameters that attempt to mend the violations. The potential function v(i, j) in Eq.(4.3) then becomes:

$$v(i,j) = \begin{cases} w_{ij}\varphi(x_i, x_j) & \text{if } c_{ij} = 1 \text{ and } y_i \neq y_j \\ w_{ij}(\varphi^{\max} - \varphi(x_i, x_j)) & \text{if } c_{ij} = -1 \text{ and } y_i = y_j \\ 0 & \text{otherwise} \end{cases}$$
(4.9)

and the objective function for semi-supervised clustering in Eq.(4.6) can be expressed as:

$$\mathcal{J}_{\text{obj}} = \sum_{x_i \in \mathcal{X}} d_{\mathcal{A}}(x_i, \mu(i)) + \sum_{\substack{(x_i, x_j) \in \mathcal{C}_{ML} \\ s.t. \ y_i \neq y_j}} w_{ij} \varphi(x_i, x_j) + \sum_{\substack{(x_i, x_j) \in \mathcal{C}_{CL} \\ s.t. \ y_i = y_j}} w_{ij} (\varphi^{\max} - \varphi(x_i, x_j)) - \log \Pr(\mathcal{A}) + n \log Z_{\Theta}$$
(4.10)

Note that the MRF partition function term  $\log Z$  has been dropped from the objective function. Its estimation cannot be performed in closed form for most non-trivial dependency structures, and while approximate inference methods could be employed for computing it (Kschischang, Frey, & Loeliger, 2001; Wainwright & Jordan, 2003), experiments with the different methods have shown that minimizing the simplified objective yields comparable results (Bilenko & Basu, 2004).

## 4.3.1 Parameter Priors

Following the definition of  $\Theta$  in Section 4.2.1, the prior term  $\log Pr(\Theta)$  in Eq.(4.6) and the subsequent equations can be factored as follows:

$$\log \Pr(\Theta) = \log \left( \Pr(\mathcal{A}) \Pr(\mathcal{M}) \right) = \log \Pr(\mathcal{A}) + P_M$$

where the distortion parameters  $\mathcal{A}$  are assumed to be independent of the cluster centroids  $\mathcal{M} = {\{\mu_i\}_{i=1}^K}$ , and uniform priors are considered over the cluster centroids (leading to the constant term  $P_M$ ). For different distortion measures, parameter values may exist that lead to degenerate solutions of the optimization problem. For example, for squared Euclidean distance, the zero matrix  $A = \mathbf{0}$  is one such solution. To prevent degenerate solutions,  $Pr(\mathcal{A})$  is used to regularize the parameter values using a prior distribution.

If the standard Gaussian prior was used on the parameters of the distortion function, it would allow the parameters to take negative values. Since it is desirable to constrain the parameter values to be non-negative, it is more appropriate to use the Rayleigh distribution (Papoulis & Pillai, 2001). Assuming independence of the parameters  $a_i \in A$ , the prior term based on the Rayleigh distribution is the following:

$$\Pr(A) = \prod_{a_i \in A} \frac{a_i \exp\left(-\frac{a_i^2}{s^2}\right)}{s^2}$$
(4.11)

where *s* is the width parameter.

Next, we consider three examples of commonly used distortion measures and their parameterizations for use with HMRF-KMEANS: squared Euclidean distance, cosine distance and KL divergence. Through learning, each of these similarity functions reflects the correct notion of similarity provided by the pairwise constraints, leading to better clustering accuracy.

## 4.3.2 Parameterized Squared Euclidean Distance

Squared Euclidean distance is parameterized using a symmetric positive-definite matrix *A* as follows:

$$d_{euc_A}(x_i, x_j) = \|x_i - x_j\|_A^2 = (x_i - x_j)^T A(x_i - x_j).$$
(4.12)

This form of the parameterized squared Euclidean distance is equivalent to Ma-

halanobis distance with an arbitrary positive semidefinite weight matrix A in place of the inverse covariance matrix, and it was previously used by (Xing, Ng, Jordan, & Russell, 2003) and (Bar-Hillel, Hertz, Shental, & Weinshall, 2003). Such formulation can also be viewed as a projection of every instance x onto a space spanned by  $A^{1/2}$ :  $x \rightarrow A^{1/2}x$ .

The  $\varphi$  function that penalizes constraint violations is defined as  $\varphi(x_i, x_j) = d_{euc_A}(x_i, x_j)$ . One possible initialization of the upper bound for cannot-link penalties is  $\varphi_{euc_A}^{\max} = \sum_{(x_i, x_j) \in C_{CL}} d_{euc_A}(x_i, x_j)$ , which guarantees that the penalty is always positive. Using these definitions in the objective in Eq.(4.10), the following objective function is obtained for semi-supervised clustering with parameterized squared Euclidean distance:

$$\mathcal{J}_{euc_A} = \sum_{x_i \in \mathcal{X}} d_{euc_A}(x_i, \mu(i)) + \sum_{\substack{(x_i, x_j) \in C_{ML} \\ s.t. \ y_i \neq y_j}} w_{ij} d_{euc_A}(x_i, x_j)$$
  
+ 
$$\sum_{\substack{(x_i, x_j) \in C_{CL} \\ s.t. \ y_i = y_j}} w_{ij} \left( \varphi_{euc_A}^{\max} - d_{euc_A}(x_i, x_j) \right) - \log \Pr(A) - n \log \det(\mathcal{A})$$
(4.13)

Note that the  $\log Z_{\Theta}$  term in the general objective function in Eq.(4.10) is computable in closed form for a Gaussian distribution with covariance matrix  $A^{-1}$ , resulting in the  $\log \det(A)$  term.

## 4.3.3 Parameterized Cosine Distance

Cosine distance can be parameterized using a symmetric positive-definite matrix *A*, which leads to the following distortion measure:

$$d_{\cos_A}(x_i, x_j) = 1 - \frac{x_i^T A x_j}{\|x_i\|_A \|x_j\|_A}.$$
(4.14)

Because for realistic high-dimensional domains computing the full matrix A is very expensive computationally, diagonal matrix is considered in this case, such that a = diag(A) is a vector of positive weights, intuitively corresponding to the relative importance of each dimension.

To use parameterized cosine distance as the adaptive distortion measure for clustering, the  $\varphi$  function is defined as  $\varphi(x_i, x_j) = d_{\cos_A}(x_i, x_j)$ . Using this definition along with Eq.(4.10), and setting  $\varphi^{\max} = 1$  as an upper bound on  $\varphi(x_i, x_j)$ , the following objective function is obtained for semi-supervised clustering with parameterized cosine distance:

$$\mathcal{J}_{\cos_{A}} = \sum_{x_{i} \in \mathcal{X}} d_{\cos_{A}}(x_{i}, \mu(i)) + \sum_{\substack{(x_{i}, x_{j}) \in \mathcal{C}_{ML} \\ s.t. \ y_{i} \neq y_{j}}} w_{ij} d_{\cos_{A}}(x_{i}, x_{j})$$
  
+ 
$$\sum_{\substack{(x_{i}, x_{j}) \in \mathcal{C}_{CL} \\ s.t. \ y_{i} = y_{j}}} w_{ij} \left(1 - d_{\cos_{A}}(x_{i}, x_{j})\right) - \log \Pr(\mathcal{A})$$
(4.15)

Note that the  $\log Z_{\Theta}$  term is difficult to compute in closed form (Banerjee et al., 2005a), so it is assumed to be constant during the clustering process and therefore dropped from the objective function. This assumption is reasonable given an appropriate prior Pr(A), and experimentally we have not observed problems with algorithm convergence due to this approximation.

## 4.3.4 Parameterized Kullback-Leibler Divergence

In domains where each instance can be described a probability distribution, Kullback-Leibler divergence can be used to measure similarity between instances. In previous work, (Cohn, Caruana, & McCallum, 2003) parameterized KL-divergence by multiplying every component by a real weight:  $d'_{KL}(x_i, x_j) = \sum_{m=1}^{d} a_m x_{im} \log \frac{x_{im}}{x_{jm}}$ .

We use a similar parameterization of KL divergence, where the vector of positive weights, *a*, corresponds to a diagonal matrix *A*. However, since after the reweighting each instance is no longer a probability distribution, this parameterization requires using I-divergence, a function that also belongs to the class of Bregman divergences (Banerjee et al., 2005b). I-divergence has the form:  $d_I(x_i, x_j) = \sum_{m=1}^d x_{im} \log \frac{x_{im}}{x_{jm}} - \sum_{m=1}^d (x_{im} - x_{jm})$ ,

where  $x_i$  and  $x_j$  no longer need to be probability distributions but can be any non-negative vectors.<sup>1</sup> The parameterized I-divergence is expressed as follows:

$$d_{I_A}(x_i, x_j) = \sum_{m=1}^d a_m x_{im} \log \frac{x_{im}}{x_{jm}} - \sum_{m=1}^d a_m (x_{im} - x_{jm}),$$
(4.16)

which can be interpreted as scaling every component of the original probability distribution by a weight contained in the corresponding component of a, and then taking I-divergence between the transformed vectors.

The HMRF framework requires defining an appropriate penalty violation function  $\varphi$  that is symmetric, since the constraint pairs are unordered. To meet this requirement, a sum of weighted I-divergences from  $x_i$  and  $x_j$  to the mean vector  $\frac{x_i+x_j}{2}$  is used. This parameterized I-divergence to the mean,  $d_{IM_A}$ , is equivalent to weighted Jensen-Shannon divergence (Cover & Thomas, 1991), the symmetric KL-divergence to the mean, and is defined as follows:

$$d_{IM_A}(x_i, x_j) = \sum_{m=1}^d a_m \left( x_{im} \log \frac{2x_{im}}{x_{im} + x_{jm}} + x_{jm} \log \frac{2x_{jm}}{x_{im} + x_{jm}} \right).$$
(4.17)

Then, defining the constraint violation function  $\varphi$  as  $\varphi(x_i, x_j) = d_{IM_A}(x_i, x_j)$  yields the following objective function for semi-supervised clustering with parameterized I-divergence:

$$\mathcal{J}_{I_{A}} = \sum_{x_{i} \in \mathcal{X}} d_{I_{A}}(x_{i}, \mu(i)) + \sum_{\substack{(x_{i}, x_{j}) \in \mathcal{C}_{ML} \\ s.t. \ y_{i} \neq y_{j}}} w_{ij} d_{IM_{A}}(x_{i}, x_{j}) \\
+ \sum_{\substack{(x_{i}, x_{j}) \in \mathcal{C}_{CL} \\ s.t. \ y_{i} = y_{j}}} w_{ij} (d_{IM_{A}}^{\max} - d_{IM_{A}}(x_{i}, x_{j})) - \log \Pr(\mathcal{A})$$
(4.18)

The upper bound  $d_{IM_A}^{\text{max}}$  can be initialized as  $d_{IM_A}^{\text{max}} = \sum_{m=1}^{d} a_m$ , which follows from <sup>1</sup>For probability distributions, I-divergence and KL-divergence are equivalent. the fact that unweighted Jensen-Shannon divergence is bounded above by 1 (Lin, 1991).

As for cosine distance, the  $\log Z_{\Theta}$  term is difficult to compute in closed form for parameterized I-divergence (Banerjee et al., 2005a), so it is assumed to be constant during the clustering process and therefore dropped from the objective function. Again, this assumption is reasonable given an appropriate prior Pr(A), and experimentally we have not observed problems with algorithm convergence due to this approximation.

# 4.4 Learning Similarity Functions within the HMRF-KMeans Algorithm

Since the cluster assignments and the generative model parameters are unknown in a clustering setting, minimizing the general objective function in Eq.(4.10) is an "incomplete-data problem". A popular solution technique for such problems is the *Expectation-Maximization* (EM) algorithm (Dempster, Laird, & Rubin, 1977). The K-Means algorithm (MacQueen, 1967) is known to be equivalent to the EM algorithm with hard clustering assignments, under certain assumptions (Kearns et al., 1997; Basu et al., 2002; Banerjee et al., 2005b). This section describes a K-Means-type hard partitional clustering algorithm, HMRF-KMEANS, that finds a local minimum of the semi-supervised clustering objective function  $\mathcal{J}_{obj}$  in Eq.(4.10).

The outline of the algorithm is presented in Fig. 4.3. The basic idea of HMRF-KMEANS is as follows. First, the constraints are used to obtain a good initialization of the cluster centroids. Then, in the E-step, given the current cluster representatives, every data point is re-assigned to the cluster which minimizes its contribution to  $\mathcal{J}_{obj}$ . In the M-step, the cluster centroids  $\mathcal{M} = {\mu_i}_{i=1}^K$  are re-estimated given current assignments to minimize  $\mathcal{J}_{obj}$  for the current assignment of points to clusters. The clustering distortion measure  $d_{\mathcal{A}}$ is subsequently updated in the M-step to reduce the objective function by modifying the parameters  $\mathcal{A}$  of the distortion measure.

Algorithm: HMRF-KMEANS
<b>Input:</b> Set of data points $X = \{x_i\}_{i=1}^n$
Set of constraints $C$
Parameterized distortion measure $d_{\mathcal{A}}(\cdot, \cdot)$ .
Constraint violation costs $\mathcal W$
Desired number of clusters K
<b>Output:</b> Disjoint <i>K</i> -partitioning $\{X_i\}_{i=1}^n$ of X such that objective
function $\mathcal{J}_{obj}$ in Eqn. (3.9) is (locally) minimized.
Method:
1. Initialize the K clusters centroids $\mathcal{M}^{(0)} = \{\mu_1^{(0)}\}_{i=1}^K$ , set t $\leftarrow 0$
2. Repeat until <i>convergence</i>
2a. E-step: Given centroids $\mathcal{M}^{(t)}$ and distortion parameters $\mathcal{A}^{(t)}$ ,
re-assign cluster labels $\mathcal{Y}^{(t+1)} = \{y_i^{(t+1)}\}_{i=1}^n$ on $\mathcal{X}$ to minimize $\mathcal{I}_{obj}$ .
2b. M-step(A): Given cluster labels $\mathcal{Y}^{(t+1)}$ and distortion parameters $\mathcal{A}^{(t+1)}$ ,
re-calculate centroids $\mathcal{M}^{(t+1)} = \{\mu_i^{(t+1)}\}_{i=1}^K$ to minimize $\mathcal{I}_{obj}$ .
2c. M-step(B): Given cluster labels $\mathcal{Y}^{(t+1)}$ and centroids $\mathcal{M}^{(t+1)}$ ,
re-estimate parameters $\mathcal{A}^{(t+1)}$ of the distortion measure to reduce $\mathcal{I}_{obj}$ .
2d. $t \leftarrow t+1$

Figure 4.3: The HMRF-KMEANS algorithm

Note that this corresponds to the generalized EM algorithm (Dempster et al., 1977; Neal & Hinton, 1998), where the objective function is reduced but not necessarily minimized in the M-step. Effectively, the E-step minimizes  $\mathcal{J}_{obj}$  over cluster assignments  $\mathcal{Y}$ , the M-step(A) minimizes  $\mathcal{J}_{obj}$  over cluster centroids  $\mathcal{M}$ , and the M-step(B) reduces  $\mathcal{J}_{obj}$  over the parameters A of the distortion measure  $d_{\mathcal{A}}$ . The E-step and the M-step are repeated until a specified convergence criterion is reached.

Detailed discussion of the initialization, E-step, and M-step(A) of the algorithm along with the proof of convergence can be found in (Basu, 2005), while in this section we focus on M-step(B) where the distortion measure parameters are updated to decrease the objective function.

For certain distortion measure parameterizations, minimization via taking partial derivatives and solving for the parameter values may be feasible, e.g., for squared Euclidean distance with uniform parameter priors (Bilenko et al., 2004), in which case the weight matrix A is obtained in M-Step(B) as:

$$A = |\mathcal{X}| \left( \sum_{\substack{x_i \in \mathcal{X} \\ x_i \in \mathcal{X}}} (x_i - \mu_{y_i}) (x_i - \mu_{y_i})^T + \sum_{\substack{(x_i, x_j) \in \mathcal{C}_{ML} \\ s.t. \ y_i \neq y_j}} w_{ij} (x_i - x_j) (x_i - x_j)^T \right)^{-1} + \sum_{\substack{(x_i, x_j) \in \mathcal{C}_{CL} \\ s.t. \ y_i = y_i}} w_{ij} \left( \sum_{\substack{(x'_i, x'_j) \in \mathcal{C}_{CL} \\ (x'_i, x'_j) \in \mathcal{C}_{CL}}} (x'_i - x'_j) (x'_i - x'_j)^T - (x_i - x_j) (x_i - x_j)^T \right)^{-1}$$
(4.19)

Since the weight matrix *A* is obtained by inverting the summation of covariance matrices in Eq.(4.19), that summation (corresponding to  $\frac{1}{|X|}\mathcal{A}^{-1}$ ) must not be singular. If at any iteration the summation is singular, it can be conditioned via adding the identity matrix multiplied by a small fraction of the trace of  $A^{-1}$ :  $A^{-1} = A^{-1} + \varepsilon \operatorname{tr}(A^{-1})\mathbf{I}$ . If the weight matrix *A* resulting from the inversion is negative definite, it is mended by projecting on the set  $C = \{A : A \succeq 0\}$  of positive semi-definite matrices, to ensure that the squared Euclidean distance parameterized by *A* is a Mahalanobis distance (Golub & van Loan, 1989).

In general, for parameterized Bregman divergences or directional distances with general parameter priors, it is difficult to obtain a closed form update for the parameters of the distortion measure that can minimize the objective function. Gradient descent provides an alternative avenue for learning the distortion measure parameters.

For squared Euclidean distance, a full parameter matrix *A* is updated during gradient descent using the rule:  $A = A + \eta \frac{\partial f_{euc_A}}{\partial A}$  (where  $\eta$  is the learning rate). Using Eq.(4.13),  $\frac{\partial f_{euc_A}}{\partial A}$  can be expressed as:

$$\frac{\partial \mathcal{J}_{euc_A}}{\partial A} = \sum_{x_i \in X} \frac{\partial d_{euc_A}(x_i, \mu(i))}{\partial A} + \sum_{\substack{(x_i, x_j) \in C_{ML} \\ s.t. \ y_i \neq y_j}} w_{ij} \frac{\partial d_{euc_A}(x_i, x_j)}{\partial A} + \sum_{\substack{(x_i, x_j) \in C_{ML} \\ s.t. \ y_i = y_j}} w_{ij} \left[ \frac{\partial \varphi_{euc_A}^{\max}}{\partial A} - \frac{\partial d_{euc_A}(x_i, x_j)}{\partial A} \right] - \frac{\partial \log \Pr(A)}{\partial A} - n \frac{\partial \log \det(A)}{\partial A}.$$
(4.20)

The gradient of the parameterized squared Euclidean distance is given by:

$$\frac{\partial d_{euc_A}(x_i, x_j)}{\partial A} = (x_i - x_j)(x_i - x_j)^T$$

The derivative of the upper bound  $\varphi_{euc_A}^{\max}$  is  $\frac{\partial \varphi_{euc_A}^{\max}}{\partial A} = \sum_{(x_i, x_j) \in C_{CL}} (x_i - x_j) (x_i - x_j)^T$  if  $\varphi_{euc_A}^{\max}$  is computed as described in Section 4.3.2. In practice, one can initialize  $\varphi_{euc_A}^{\max}$  with a sufficiently large constant, which would make its derivative zero. Accordingly, an extra condition must be then inserted into the algorithm to guarantee that penalties for violated cannot-link constraints are never negative, in which case the constant must be increased.

When Rayleigh priors are used on the set of parameters A, the partial derivative of the log-prior with respect to every individual parameter  $a_m \in A$ ,  $\frac{\partial \log \Pr(A)}{\partial a_m}$ , is given by:

$$\frac{\partial \log \Pr(A)}{\partial a_m} = \frac{1}{a_m} - \frac{a_m}{s^2}$$
(4.21)

The gradient of the distortion normalizer  $\log \det(A)$  term is as follows:

$$\frac{\partial \log \det(A)}{\partial A} = 2A^{-1} - diag(A^{-1}). \tag{4.22}$$

For parameterized cosine distance and KL divergence, a diagonal parameter matrix *A* is considered, where a = diag(A) is a vector of positive weights. During gradient descent, each weight  $a_m$  is updated individually:  $a_m = a_m + \eta \frac{\partial f_{obj}}{\partial a_m}$  ( $\eta$  is the learning rate). Using Eq.(4.10),  $\frac{\partial f_{obj}}{\partial a_m}$  can be expressed as:

$$\frac{\partial \mathcal{J}_{\text{obj}}}{\partial a_m} = \sum_{x_i \in X} \frac{\partial d_{\mathcal{A}}(x_i, \mu(i))}{\partial a_m} + \sum_{\substack{(x_i, x_j) \in C_{ML} \\ s.t. \ y_i \neq y_j}} w_{ij} \frac{\partial \varphi(x_i, x_j)}{\partial a_m} + \sum_{\substack{(x_i, x_j) \in C_{CL} \\ s.t. \ y_i = y_j}} w_{ij} \left[ \frac{\partial \varphi^{\max}}{\partial a_m} - \frac{\partial \varphi(x_i, x_j)}{\partial a_m} \right] - \frac{\partial \log \Pr(A)}{\partial a_m}$$
(4.23)

The gradients of the corresponding distortion measures and constraint potential functions for parameterized cosine distance and KL divergence are the following:

$$\frac{\partial d_{\cos_A}(x_i, x_j)}{\partial a_m} = \frac{x_{im} x_{jm} \|x_i\|_A \|x_j\|_A - x_i^T A x_j \frac{x_{im}^2 \|x_j\|_A^2 + x_{jm}^2 \|x_i\|_A^2}{2\|x_i\|_A \|x_j\|_A}}{\|x_i\|_A^2 \|x_j\|_A^2}, 
\frac{\partial d_{I_A}(x_i, x_j)}{\partial a_m} = x_{im} \log \frac{x_{im}}{x_{jm}} - (x_{im} - x_{jm}), 
\frac{\partial d_{IM_A}(x_i, x_j)}{\partial a_m} = x_{im} \log \frac{2x_{im}}{x_{im} + x_{jm}} + x_{jm} \log \frac{2x_{jm}}{x_{im} + x_{jm}},$$
(4.24)

while the gradient of the upper bound  $\frac{\partial \varphi^{\max}}{\partial a_m}$  is 0 for parameterized cosine and 1 for parameterized KL divergence, as follows from the expressions for these constants in Sections 4.3.3 and 4.3.4.

For all distortion metrics, individual similarity function parameters can be learned for each cluster, allowing the clusters to lie in different subspaces. To implement clusterspecific similarity function learning, the above updates should be based only on points belonging to the cluster, ignoring the rest of the dataset.

Overall, the distance learning step results in modifying the distortion measure parameters so that data points in violated must-link constraints are brought closer together, while points in violated cannot-link constraints are pulled apart, and each dimension is scaled proportionally to data variance. This process leads to a transformed data space that facilitates partitioning of the unlabeled data by attempting to mend the constraint violations while capturing the natural variance in the data.

# 4.5 **Experimental Results**

This section describes the experiments that were performed to demonstrate the effectiveness of using learnable similarity functions within HMRF-KMEANS.

## 4.5.1 Datasets

Experiments were run on both low-dimensional and high-dimensional datasets to evaluate the HMRF-KMEANS framework with different distortion measures. For the lowdimensional datasets, on which squared Euclidean distance was used as the distortion measure, the following datasets were considered:

- Three datasets from the UCI repository: *Iris, Wine,* and *Ionosphere* (Blake & Merz, 1998);
- The Protein dataset used by Xing et al. (2003) and Bar-Hillel et al. (2003);
- Randomly sampled subsets from the *Digits* and *Letters* handwritten character recognition datasets, also from the UCI repository. For *Digits* and *Letters*, two sets of three classes were chosen: {I, J, L} from *Letters* and {3, 8, 9} from *Digits*, sampling 10% of the data points from the original datasets randomly. These classes were chosen since they represent difficult visual discrimination problems.

Table 4.1 summarizes the properties of the low-dimensional datasets: the number of instances, the number of dimensions, and the number of classes.

	Iris	Wine	Ionosphere	Protein	Letters	Digits
Instances	150	178	351	116	227	317
Dimensions	4	13	34	20	16	16
Classes	3	3	2	6	3	3

Table 4.1: Low-dimensional datasets used in experimental evaluation

For the high-dimensional text data, 3 datasets that have the characteristics of being sparse, high-dimensional, and having a small number of points compared to the dimensionality of the space were considered. This is done for two reasons:

- When clustering sparse high-dimensional data, e.g., text documents represented using the vector space model, it is particularly difficult to cluster small datasets, as observed by Dhillon and Guan (2003). The purpose of performing experiments on these subsets is to scale down the sizes of the datasets for computational reasons but at the same time not scale down the difficulty of the tasks.
- Clustering small number of sparse high-dimensional data points is a likely scenario in realistic applications. For example, when clustering the search results in a websearch engine like Vivísimo<sup>2</sup>, the number of webpages that are being clustered is typically in the order of hundreds. However, the dimensionality of the feature space, corresponding to the number of unique words in all the webpages, is in the order of thousands. Moreover, each webpage is sparse, since it contains only a small number of all the possible words. On such datasets, clustering algorithms can easily get stuck in local optima: in such cases it has been observed that there is little relocation of documents between clusters for most initializations, which leads to poor clustering quality after convergence of the algorithm (Dhillon & Guan, 2003). Supervision in the form of pairwise constraints is most beneficial in such cases and may significantly improve clustering quality.

Three datasets were derived from the 20-Newsgroups collection.<sup>3</sup> This collection has messages harvested from 20 different Usenet newsgroups, 1000 messages from each newsgroup. From the original dataset, a reduced dataset was created by taking a random subsample of 100 documents from each of the 20 newsgroups. Three datasets were created by selecting 3 categories from the reduced collection. News-Similar-3 consists of 3 newsgroups on similar topics (comp.graphics, comp.os.ms-windows, comp.windows.x) with significant overlap between clusters due to cross-posting. News-Related-3 consists of 3 newsgroups on related topics (talk.politics.misc, talk.politics.guns, and

<sup>&</sup>lt;sup>2</sup>http://www.vivisimo.com

<sup>&</sup>lt;sup>3</sup>http://www.ai.mit.edu/people/jrennie/20Newsgroups

talk.politics.mideast). *News-Different-3* consists of articles posted in 3 newsgroups that cover different topics (alt.atheism, rec.sport.baseball, sci.space) with well-separated clusters. All the text datasets were converted to the vector-space model by tok-enization, stop-word removal, TF-IDF weighting, and removal of very high-frequency and low-frequency words, following the methodology of Dhillon and Modha (2001). Table 4.2 summarizes the properties of the high-dimensional datasets.

Table 4.2: High-dimensional datasets used in experimental evaluation

	News-Different-3	News-Related-3	News-Similar-3
Instances	300	300	300
Dimensions	3251	3225	1864
Classes	3	3	3

# 4.5.2 Clustering Evaluation

*Normalized mutual information* (NMI) was used as the clustering evaluation measure. NMI is an external clustering validation metric that estimates the quality of the clustering with respect to a given underlying class labeling of the data: it measures how closely the clustering algorithm could reconstruct the underlying label distribution in the data (Strehl, Ghosh, & Mooney, 2000). If  $\hat{Y}$  is the random variable denoting the cluster assignments of the points and *Y* is the random variable denoting the underlying class labels on the points, then the NMI measure is defined as:

$$NMI = \frac{I(Y;\hat{Y})}{(H(Y) + H(\hat{Y}))/2}$$
(4.25)

where I(X;Y) = H(X) - H(X|Y) is the mutual information between the random variables *X* and *Y*, H(X) is the Shannon entropy of *X*, and H(X|Y) is the conditional entropy of *X* given *Y* (Cover & Thomas, 1991). NMI effectively measures the amount of statistical information shared by the random variables representing the cluster assignments and the user-labeled class assignments of the data points. Though various clustering evaluation measures have been used in the literature, NMI and it's variants have become popular lately among clustering practitioners (Dom, 2001; Fern & Brodley, 2003; Meila, 2003).

## 4.5.3 Methodology

Learning curves were generated using two-fold cross-validation performed over 20 runs on each dataset. In every trial, 50% of the dataset was set aside as the training fold. Every point on the learning curve corresponds to the number of constraints on pairs of data points from the training fold. These constraints are obtained by randomly selecting pairs of points from the training fold and creating must-link or cannot-link constraints depending on whether the underlying classes of the two points are the same or different. Unit constraint costs *W* were used for all constraints (original and inferred), since the datasets did not provide individual weights for the constraints. The gradient step size  $\eta$  for learning the distortion measure parameters and the Rayleigh prior width parameter *s* were set based on pilot studies. The gradient step size was set to  $\eta = 100.0$  for clustering with weighted cosine distance  $d_{\cos_A}$ and  $\eta = 0.08$  for weighted I divergence  $d_{I_A}$ . The Rayleigh prior width parameter was set to s = 1. In a real-life setting, the free parameters of the algorithm could be tuned using cross-validation with a hold-out set. The clustering algorithm was run on the whole dataset, but NMI was calculated using points in the test fold.

Sensitivity experiments were performed with HMRF-KMEANS to study the effectiveness of employing learnable similarity functions. The proposed HMRF-KMEANS algorithm was compared with three ablations, as well as with unsupervised K-Means clustering. The following variants were compared for distortion measures  $d_{\cos_A}$ ,  $d_{I_A}$  and  $d_{euc_A}$ :

HMRF-KMEANS-C-D-R is the complete HMRF-KMEANS algorithm that incorporates constraints in cluster assignments (C), performs distortion measure learning (D), and also performs regularization (R) using a Rayleigh prior as described in Section 4.3.1;

- HMRF-KMEANS-C-D is the first ablation of HMRF-KMEANS that includes all components except for regularization of distortion measure parameters;
- HMRF-KMEANS-C is an ablation of HMRF-KMEANS that uses pairwise supervision for initialization and cluster assignments, but does not perform distortion measure learning;
- RCA-KMEANS is K-Means algorithm that uses distortion measure parameters learned using the Relevant Components Analysis (RCA) algorithm of Bar-Hillel et al. (2003);
- KMEANS is the unsupervised K-Means algorithm;

The goal of these experiments was to evaluate the utility of distortion measure learning HMRF framework and identify settings in which particular components are beneficial. For low-dimensional datasets, we also compared several distinct possibilities for parameterizing the distance metric  $d_{euc_A}$ :

- HMRF-KMEANS-C-D-R is the complete HMRF-KMEANS algorithm that learns a single diagonal weight matrix for the entire dataset (*A* is diagonal and identical for all clusters);
- HMRF-KMEANS-C-D-R-M is the complete HMRF-KMEANS algorithm that learns *K* diagonal weight matrices  $A_i, \ldots, A_k$  so that each cluster corresponds to a distinct similarity function;
- HMRF-KMEANS-C-D-R-FULL is the complete HMRF-KMEANS algorithm that learns a single fully-parameterized Mahalanobis distance: *A* is a *d* × *d* positive-definite matrix that is identical for all clusters.

The goal of these experiments is to study the utility of learning a full parameterization of the similarity function (effectively training a Mahalanobis distance) versus only using a diagonal parameterization (learning weights for a Euclidean distance), since the latter is significantly cheaper computationally. Results obtained with learning individual similarity functions for each cluster illustrate the utility of allowing different clusters to lie in different subspaces, as opposed to learning a single set of similarity function parameters for the entire dataset.

### 4.5.4 Results and Discussion

**Low-dimensional datasets:** Figures 4.4-4.15 show learning curves for the ablation experiments on the six low-dimensional datasets. Across all datasets, the overall HMRF-KMEANS approach without regularization (KMEANS-C-D) outperforms the constraints-only ablation and unsupervised KMeans. Since the performance of KMEANS-C-D-R is not substantially different from KMEANS-C-D, it can be concluded that regularization does not lead to performance improvements on low-dimensional datasets. This can be explained by the fact that the number of distortion measure parameters is small for low-dimensional domains while estimates obtained from data do not have high variance, and therefore incorporating a prior in the probabilistic model is not necessary.

For the *Wine*, *Protein*, and *Digits-389* datasets, the difference between ablations that utilize metric learning (KMEANS-C-D-R and KMEANS-C-D) and those that do not (KMEANS-C and KMEANS) at the beginning of the learning curve indicates that even in the absence of constraints, weighting features by their variance (essentially using unsupervised Mahalanobis distance) improves clustering accuracy. For the *Wine* dataset, additional constraints provide an insubstantial improvement in cluster quality on this dataset, which shows that meaningful feature weights are obtained from scaling by variance using just the unlabeled data.

Comparing the performance of different variants of HMRF-KMEANS with *RCA*, we can see that the ability to embed similarity function learning within the clustering algorithm leads to significantly better results for HMRF-KMEANS. This is explained by



Figure 4.5: Results for  $d_{euc}$  on the *Iris* dataset with full and per-cluster parameterizations







Figure 4.7: Results for  $d_{euc}$  on the *Wine* dataset with full and per-cluster parameterizations







Figure 4.9: Results for  $d_{euc}$  on the *Protein* dataset with full and per-cluster parameterizations





Figure 4.11: Results for  $d_{euc}$  on the *Ionosphere* dataset with full and per-cluster parameterizations





Figure 4.12: Results for  $d_{euc}$  on the Digits-389 dataset

Figure 4.13: Results for  $d_{euc}$  on the *Digits-389* dataset with full and per-cluster parameterizations



Figure 4.14: Results for  $d_{euc}$  on the Letters-IJL dataset



Figure 4.15: Results for  $d_{euc}$  on the *Letters-IJL* dataset with full and per-cluster parameterizations

the fact that *RCA* utilizes only the pairwise constraints for learning the similarity function parameters, while HMRF-KMEANS uses both the constraints and the unlabeled data, adjusting the parameters gradually in the course of clustering.

The results for learning full-matrix and per-cluster parameterizations of the similarity function demonstrate that both of these extensions can lead to significant improvements in clustering quality. However, the relative usefulness of these two techniques varies between the datasets. Multiple similarity functions are beneficial for all datasets except for *Protein* where they did not affect the results, and *Iris*, where they had a negative effect. Using the full matrix parameterization also did not affect *Protein* results, and had a negative effect on *Digits*, while it improved results on the other four datasets. This inconsistency can be explained by the fact that the relative success of the two techniques depends on the properties of a particular dataset: using a full weight matrix helps when the features are highly correlated, while using per-cluster parameterization lead to improvements when clusters in the dataset are of different shapes or lie in different subspaces of the original space. A combination of the two techniques is most helpful when both of these requirements are satisfied, as for *Wine* and *Letters*, which was observed by visualizing low-dimensional projections of these datasets. For other datasets with the exception of *Protein*, either per-cluster parameterization or the full weight matrix lead to maximum performance in isolation.

Some of the HMRF-KMEANS learning curves display a characteristic "dip", where clustering accuracy decreases as a few initial constraints are provided, but after a certain point starts to increase and eventually rises above the initial point on the learning curve. One possible explanation of this phenomenon is overfitting: having just a few constraints provides unreliable supervision, forcing the algorithm to converge to inferior local optima, while increasing the number of provided constraints allows overcoming this effect. Overall, when both constraints and distortion measure learning are utilized, the unified approach benefits from the individual strengths of the two methods, as can be seen from the KMEANS-C-D results.



Figure 4.16: Results for  $d_{\cos_A}$  on the *News-Different-3* dataset



Figure 4.17: Results for  $d_{I_A}$  on the *News-Different-3* dataset



Figure 4.18: Results for  $d_{\cos_A}$  on the *News-Related-3* dataset



Figure 4.19: Results for  $d_{I_A}$  on the *News-Related-3* dataset



Figure 4.20: Results for  $d_{\cos_A}$  on the *News-Similar-3* dataset



Figure 4.21: Results for  $d_{I_A}$  on the *News-Similar-3* dataset

**High-dimensional datasets:** Figures 4.16, 4.18 and 4.20 present the results for the ablation experiments where weighted cosine similarity  $d_{\cos_A}$  was used as the distortion measure, while Figures 4.17, 4.19 and 4.21 summarize experiments where weighted I divergence  $d_{I_A}$  was used.

As the results demonstrate, the full HMRF-KMEANS algorithm with regularization (KMEANS-C-D-R) outperforms the unsupervised K-Means baseline as well as the ablated versions of the algorithm for both distortion measures  $d_{\cos_A}$  and  $d_{I_A}$ . As can be seen from results for zero pairwise constraints in Figs. 4.16-4.21, distortion measure learning is beneficial even in the absence of any pairwise constraints, since it allows capturing the relative importance of the different attributes in the unsupervised data. In the absence of supervised data or when no constraints are violated, distance learning attempts to minimize the objective function by adjusting the weights given the distortion between the unsupervised datapoints and their corresponding cluster representatives.

For high-dimensional datasets, regularization is clearly beneficial to performance, as can be seen from the improved performance of KMEANS-C-D-R over KMEANS-C-D on all datasets. This can be explained by the fact that the number of distortion measure parameters is large for high-dimensional datasets, and therefore algorithm-based estimates of parameters tend to be unreliable unless they incorporate a prior.

Overall, the experimental results demonstrate that learning similarity functions within the HMRF-KMEANS algorithm lead to significant improvements in clustering accuracy, effectively exploiting both supervision in the form of pairwise constraints and the unsupervised data.

# 4.6 Related Work

Several semi-supervised clustering approaches were proposed that incorporate adaptive distortion measures, including parameterizations of Jensen-Shannon divergence (Cohn et al., 2003) as well as Euclidean and Mahalanobis distances (Klein, Kamvar, & Manning, 2002; Bar-Hillel et al., 2003; Xing et al., 2003). These techniques use only constraints to learn the distortion measure parameters and ignore unlabeled data in the parameter learning step, as well as separate training of the similarity function from the clustering process.

In contrast, the HMRF model provides an integrated framework which incorporates *both* learning the distortion measure parameters and constraint-sensitive cluster assignments. In HMRF-KMEANS, the parameters of the similarity function are learned iteratively as the clustering progresses, utilizing both unlabeled data and pairwise constraints. The parameters are modified to decrease the parameterized distance between violated mustlinked constraints and increase it between violated cannot-link constraints, while allowing constraint violations if they accompany a more cohesive clustering.

# 4.7 Chapter Summary

This chapter has demonstrated the utility of learnable similarity functions in semi-supervised clustering, and presented a general approach for employing them within a general probabilistic framework based on Hidden Markov Random Fields (HMRFs). The framework accommodates a broad class of similarity functions (Bregman divergences), as well as directional measures such as cosine distance, making it applicable to a wide variety of domains.

The framework yields an EM-style clustering algorithm, HMRF-KMEANS, that maximizes the joint probability of observed data points, their cluster assignments, and distortion measure parameters. The fact that the similarity functions are trained within the clustering algorithm allows utilizing both labeled and unlabeled data in learning similarity function parameters, which leads to results that are superior to learning similarity functions in isolation.
# **Chapter 5**

# Learnable Similarity Functions in Blocking

In this chapter, we show how learnable similarity functions can be employed not only for improving the accuracy of tasks that rely on pairwise similarity computations, but also for improving their scalability. We introduce an adaptive framework for learning blocking functions that are efficient and accurate for a given domain by automatically constructing them from combinations of blocking predicates. Our approach allows formulating this task as an instance of the Red-Blue Set Cover problem, approximation algorithms for which can be used for learning blocking functions.

# 5.1 Motivation

As discussed in Section 2.4, intelligent data analysis tasks that rely on computing pairwise similarities require blocking methods for scaling up to large datasets due to the quadratic number of instance pairs in a given dataset. Manual selection of fields and parameter tuning are required by all existing blocking strategies to reduce the number of returned dissimilar pairs while retaining the similar pairs.

Since an appropriate blocking strategy can be highly domain-dependent, the ad-hoc construction and manual tuning of blocking methods is difficult. They may lead to overselection of many dissimilar pairs which impedes efficiency, or, worse, under-selection of important similar pairs which decreases accuracy. Because there can be many potentially useful blocking criteria over multiple object attributes, there is a need for automating the process of constructing blocking strategies so that all or nearly all same-entity or same-cluster pairs are retained while the maximum number of dissimilar pairs is discarded.

In subsequent sections, we formalize the problem of learning an optimal blocking strategy using training data. In many record linkage domains, some fraction of instances contains true entity identifiers, e.g., UPC (bar code) numbers for retail products, SSN numbers for individuals, or DOI identifiers for citations. Presence of such labeled data allows evaluating possible blocking functions and selecting from them one that is optimal, that is, one that selects all or nearly all positive record pairs (that refer to the same entity), and a minimal number of negative pairs (that refer to different entities).

We propose to construct blocking functions based on sets of general *blocking predicates* which efficiently select all instance pairs that satisfy some binary similarity criterion. Figure 5.1 contains examples of predicates for specific record fields in different domains. We formulate the problem of learning an optimal blocking function as the task of finding a combination of blocking predicates that captures all or nearly all coreferent object pairs and a minimal number of non-coreferent pairs. Our approach is general in the sense that we do not place restrictions on the similarity predicates computed on instance pairs selected by blocking, such as requiring them to be an inner product or to correspond to a distance metric.

Domain	Blocking Predicate		
Census Data	Same 1 <sup>st</sup> Three Chars in Last Name		
Product Normalization	Common token in Manufacturer		
Citations	Publication Year same or off-by-one		

Figure 5.1: Examples of blocking functions from different record linkage domains

We consider two types of blocking functions: (1) disjunctions of blocking predicates, and (2) predicates combined in disjunctive normal form (DNF). While finding a globally optimal solution for these formulations is NP-hard, we describe an effective approximation method for them and discuss implementation issues. Empirical evaluation on synthetic and real-world record linkage datasets demonstrates the efficiency of our techniques.

### 5.2 Adaptive Blocking Formulation

Let us formally define the problem of learning an optimal blocking function. We assume that a training dataset  $\mathcal{D}_{train} = \{X, \mathcal{Y}\}$  is available that includes a set  $X = \{x_i\}_{i=1}^n$  of *n* records known to refer to *m* true objects:  $\mathcal{Y} = \{y_i\}_{i=1}^n$ , where each  $y_i$  is the true object identifier for the *i*-th record:  $y_i \in \{1, \dots, m\}$ . Each record  $x_i$  may have one or more fields.

We assume that a set of *s* general *blocking predicates*  $\{p_i\}_{i=1}^s$  is available, where each predicate  $p_i$  corresponds to two functions:

- Indexing function h<sub>i</sub>(·) is a unary function that is applied to a field value from some domain Dom(h<sub>i</sub>) (e.g., strings, integers, or categories) and generates one or more keys for the field value: h<sub>i</sub> : Dom(h<sub>i</sub>) → U<sup>\*</sup>, where U is the set of all possible keys;
- *Equality function* p<sub>i</sub>(·, ·) returns 1 if the intersection of the key sets produced by the indexing function on its arguments is non-empty, and returns zero otherwise: p<sub>i</sub>(x<sub>j</sub>, x<sub>k</sub>) = 1 iff h<sub>i</sub>(x<sub>j</sub>) ∩ h<sub>i</sub>(x<sub>k</sub>) ≠ Ø. Any pair (x<sub>j</sub>, x<sub>k</sub>) for which p<sub>i</sub>(x<sub>j</sub>, x<sub>k</sub>) = 1 is *covered* by the predicate p<sub>i</sub>.

Each general blocking predicate can be instantiated for a particular field (or a combination of fields) in a given domain, resulting in several *specific* blocking predicates for the domain. Given a database with d fields and a set of s general blocking predicates, we obtain  $t \le s \times d$  specific predicates  $\mathcal{P} = \{p_i\}_{i=1}^t$  by applying the general predicates to all fields of Sample record:

author	year	title	venue	other
Freund, Y.	(1995).	Boosting a weak learning algorithm by majority.	Information and Computation,	121(2), 256-285

Blocking predicates and key sets produced by their indexing functions for the record:

	author	title	venue	year	other
Contain Common Token	{freund, y}	{boosting, a, weak, learning, algorithm, by, majority}	{information, computation}	{1995}	{121, 2, 256, 285}
Exact Match	{'freund y'}	{'boosting a weak learning algorithm by majority'}	{'information and computation'}	{`1995`}	{`121 2 256 285`}
Same 1 <sup>st</sup> Three Chars	$\{fre\}$	$\{boo\}$	$\{inf\}$	<i>{199}</i>	{121}
Contains Same or Off-By-One Integer	0	0	0	{1994 <b>_</b> 1995, 1995_1996}	{120_121, 121_122, 1_2, 2_3, 255_256, 256_257, 284_285, 285_286}

Figure 5.2: Blocking key values for a sample record

the appropriate type. For example, suppose we have four general predicates defined for all textual fields: "*Contain Common Token*", "*Exact Match*", and "*Same* 1<sup>st</sup> *Three Chars*", "*Contains Same of Off-By-One Integer*". When these general predicates are instantiated for the bibliographic citation domain with five textual fields, *author, title, venue, year*, and *other*, we obtain  $5 \times 4 = 20$  specific blocking predicates for this domain. Figure 5.2 demonstrates the values produced by the indexing functions of these specific blocking predicates on a sample citation record (we assume that all strings are converted to lower-case and punctuation is removed before the application of the indexing functions):

Multiple blocking predicates are combined by an overall *blocking function*  $f_{\mathcal{P}}$  constructed using the set  $\mathcal{P}$  of predicates. Like the individual predicates,  $f_{\mathcal{P}}$  corresponds to an

indexing function that can be applied to any record, and an equality function for any pair of records. Pairs for which this equality function returns 1 are *covered*: they comprise the set of candidate pairs returned for subsequent similarity computation, while pairs for which the blocking function returns 0 are ignored (uncovered). Efficient generation of the set of candidate pairs requires computing the indexing function for all records, followed by retrieval of all candidate pairs using inverted indices.

Given the set  $\mathcal{P} = \{p_i\}_{i=1}^t$  containing *t* specific blocking predicates, the objective of the adaptive blocking framework is to identify an optimal blocking function  $f_{\mathcal{P}}^*$  that combines all or a subset of the predicates in  $\mathcal{P}$  so that the set of candidate pairs it returns contains all or nearly all coreferent (positive) record pairs and a minimal number of noncoreferent (negative) record pairs.

Formally, this objective can be expressed as follows:

$$f_{\mathcal{P}}^{*} = \underset{f_{\mathcal{P}}}{\operatorname{argmin}} \sum_{(x_{i}, x_{j}) \in \mathcal{R}} f_{\mathcal{P}}(x_{i}, x_{j})$$
s.t.  $|\mathcal{B}| - \sum_{(x_{i}, x_{j}) \in \mathcal{B}} f_{\mathcal{P}}(x_{i}, x_{j}) < \varepsilon$ 
(5.1)

where  $\Re = \{(x_i, x_j) : y_i \neq y_j\}$  is the set of non-coreferent pairs,  $\mathcal{B} = \{(x_i, x_j) : y_i = y_j\}$  is the set of coreferent pairs, and  $\varepsilon$  is a small value indicating that up to  $\varepsilon$  coreferent pairs may remain uncovered, thus accommodating noise and particularly difficult coreferent pairs. The optimal blocking function  $f_{\mathcal{P}}^*$  must be found in a hypothesis space that corresponds to some method of combining the individual blocking predicates. In this paper, we consider two classes of blocking functions:

• **Disjunctive blocking** selects record pairs that are covered by at least one blocking predicate from the subset of predicates that comprise the blocking function. This strategy can be viewed as covering pairs for which a the equality function for at least one of the selected predicates returns 1. The blocking function is trained by selecting

a subset of blocking predicates from  $\mathcal{P}$ .

• **Disjunctive Normal Form (DNF) blocking** selects object pairs that are covered by at least one conjunction of blocking predicates from a constructed set of conjunctions. This strategy can be viewed as covering record pairs for which at least one equality function of a conjunction of predicates returns 1. The blocking function is trained by constructing a DNF formula from the blocking predicates.

Each type of blocking functions leads to a distinct formulation of the objective (5.1), and we consider them individually in the following subsections.

#### 5.2.1 Disjunctive blocking

Given a set of specific blocking predicates  $\mathcal{P} = \{p_i\}_{i=1}^t$ , a disjunctive blocking function corresponds to selecting some subset of predicates  $\mathcal{P}' \subseteq \mathcal{P}$ , performing blocking using each  $p_i \in \mathcal{P}'$ , and then selecting record pairs that share at least one common key in the key sets computed by the indexing functions of the selected predicates. Thus, the equality function for the disjunctive blocking function based on subset  $\mathcal{P}' = \{p_{i_1}, \dots, p_{i_k}\}$  of predicates returns 1 if the equality function for at least one predicate returns 1:  $f_{\mathcal{P}'}(x_i, x_j) =$  $[p_{i_1}(x_i, x_j) + \dots + p_{i_k}(x_i, x_j)]$  where  $[\pi] = 1$  if  $\pi > 0$ , and 0 otherwise. If the equality function for the overall blocking function  $f_{\mathcal{P}'}$  returns 1 for a pair  $(x_i, x_j)$ , we say that this pair is *covered* by the blocking function.

Learning the optimal blocking function  $f_{\mathcal{P}}^*$  requires selecting a subset  $\mathcal{P}^*$  of predicates that results in all or nearly all coreferent pairs being covered by at least one predicate in  $\mathcal{P}^*$ , and a minimal number of non-coreferent pairs being covered. Then the general adaptive blocking problem in Eq.(5.1) can be written as follows:



Figure 5.3: Red-blue Set Cover view of disjunctive blocking

$$\boldsymbol{w}^{*} = \underset{\boldsymbol{w}}{\operatorname{argmin}} \sum_{(x_{i}, x_{j}) \in \mathcal{R}} \llbracket \boldsymbol{w}^{T} \boldsymbol{p}(x_{i}, x_{j}) > 0 \rrbracket$$
  
s.t.  $|\mathcal{B}| - \sum_{(x_{i}, x_{j}) \in \mathcal{B}} \llbracket \boldsymbol{w}^{T} \boldsymbol{p}(x_{i}, x_{j}) > 0 \rrbracket < \varepsilon$  (5.2)

w is binary

where  $\boldsymbol{w}$  is a binary vector of length t encoding which of the blocking predicates are selected as a part of  $f_{\mathcal{P}}^*$ , and  $\boldsymbol{p}(x_i, x_j) = [p_1(x_i, x_j), \dots, p_t(x_i, x_j)]^T$  is a vector of binary values returned by equality functions of the t predicates for the pair  $(x_i, x_j)$ .

This formulation of the learnable blocking problem is equivalent to the *Red-Blue Set Cover* problem if  $\varepsilon = 0$  (Carr, Doddi, Konjevod, & Marathe, 2000). Figure 5.3 illustrates the equivalence. The task of selecting a subset of predicates is represented by a graph with three sets of vertices. The bottom row of  $\beta$  vertices corresponds to positive (coreferent) record pairs designated as the set of *blue* elements  $\mathcal{B} = \{b_1, \dots, b_\beta\}$ . The top row of  $\rho$  vertices corresponds to negative (non-coreferent) record pairs designated as the set of *red* elements  $\mathcal{R} = \{r_1, \dots, r_\rho\}$ . The middle row of t vertices represents the set of blocking predicates  $\mathcal{P}$ , where each  $p_i \in \mathcal{P}$  corresponds to a set covering some red and blue elements. Every edge between an element vertex and a predicate vertex indicates that the record pair represented by the element vertex is covered by the predicate. Learning the optimal disjunctive blocking function is then equivalent to selecting a subset of predicate vertices with their incident edges so that at least  $\beta - \epsilon$  blue (positive) vertices have at least one incident edge, while the *cover cost*, equal to the number of red (negative) vertices with at least one incident edge, is minimized.

#### 5.2.2 DNF Blocking

In some domains, a disjunctive combination of blocking predicates may be an insufficient representation of the optimal blocking strategy. For example, in US Census data, conjunctions of predicates such as *"Same Zipcode AND Same 1st Char in Surname"* yield useful blocking criteria (Winkler, 2005). To incorporate such blocking criteria, we must extend the disjunctive formulation described above to a formulation based on combining predicates in disjunctive normal form (DNF). Then, the hypothesis space for the blocking function must include disjunctions of not just individual blocking predicates, but also of their conjunctions.

A search for the optimal DNF blocking function can be viewed as solving an extended variant of the red-blue set cover problem. In that variant, the cover is constructed using not only the sets representing the original predicates, but also using additionally constructed sets representing predicate conjunctions. Because the number of all possible conjunctions is exponential, only conjunctions up to fixed length k are considered. In Figure 5.3, considering a conjunction of blocking predicates corresponds to adding a vertex to the middle row, with edges connecting it to the red and blue vertices present in the intersection of covered vertex sets for the individual predicates in the conjunction.

The learnable blocking problem based on DNF blocking functions is then equivalent to constructing a set of conjunctions followed by selection of a set of predicate and conjunction vertices so that at least  $\beta - \epsilon$  positive (blue) vertices have at least one incident edge, while the cost, equal to the number of negative (red) nodes with at least one incident edge, is minimized.

# 5.3 Algorithms

#### 5.3.1 Pairwise Training Data

For clustering settings, supervision corresponds to sets of must-link (same-cluster) and cannot-link (different-cluster) pairs. For record linkage, supervision is available in many domains in the form of records for which the true entities to which they refer are known, as discussed in Section 5.1. Such labeled records comprise the training dataset  $\mathcal{D}_{train} = \{X, \mathcal{Y}\}$  that can be used to generate the pairwise supervision for learning the blocking function in the form of coreferent (positive) and non-coreferent (negative) record pairs. For large databases, it is impractical to explicitly generate and store in memory all positive pairs and negative pairs. However, the set of covered pairs for each predicate can be computed using the indexing function of the predicate to form an inverted index based on the key values returned by the indexing function. Then, bit arrays can be used to store the cover of each predicate, obtained by iteration over the inverted index.

If training data is unavailable, it can be obtained automatically by performing linkage or clustering without blocking, and then using the linkage or clustering results as training data for learning a blocking function for the given domain.

#### 5.3.2 Learning Blocking Functions

#### **Disjunctive Blocking**

The equivalence of learning optimal disjunctive blocking and the red-blue set cover problem described in Section 5.2.1 has discouraging implications for the practitioner. The red-blue set cover problem is NP-hard, and Carr et al. (2000) have shown that unless P=NP, it cannot be efficiently approximated within a factor  $O(2^{\log^{1-\delta}t})$ ,  $\delta = 1/\log \log^c t$ , where *t* is the number of predicates under consideration. On the other hand, several approximate algorithms have been proposed for the red-blue set cover problem (Carr et al., 2000; Peleg, 2000). We base our approach on a modified version of Peleg's greedy algorithm that has an approxi-

Algorithm: APPROXRBSETCOVER **Input:** Training set  $\mathcal{B} = \{b_1, \ldots, b_\beta\}$  and  $\mathcal{R} = \{r_1, \ldots, r_\rho\}$  where each  $b_i \in \mathcal{B}$  is a pair of coreferent records  $(x_{i_1}, x_{i_2})$  s.t.  $y_{i_1} = y_{i_2}$ each  $r_i \in \mathcal{R}$  is a pair of non-coreferent records  $(x_{i_1}, x_{i_2})$  s.t.  $y_{i_1} \neq y_{i_2}$ Set of blocking predicates  $\mathcal{P} = \{p_1, \dots, p_t\}$ Maximum number of coreferent pairs allowed to be uncovered  $\varepsilon$ Maximum number of pairs that any predicate may cover  $\eta$ **Output:** A disjunctive blocking function based on subset  $\mathcal{P}^* \subset \mathcal{P}$ Method: 1. Discard from  $\mathcal{P}$  all predicates  $p_i$  for which  $r(p_i) > \eta$ :  $\mathcal{P} \leftarrow \{ p_i \in \mathcal{P} | r(p_i) \leq \eta \}.$ 2. If  $|\mathcal{B}| - |\mathcal{B}(\mathcal{P})| > \varepsilon$  return  $\mathcal{P}$  (cover is not feasible,  $\eta$  is too low) 3. Set  $\gamma = \sqrt{t/\log\beta}$ . 4. Discard all  $r_i$  covered by more than  $\gamma$  predicates:  $\mathcal{R} \leftarrow \{r_i \in \mathcal{R} | \deg(r_i, \mathcal{P}) \leq \gamma\}$ 5. Construct an instance of weighted set cover  $\mathcal{T}$  by discarding elements of  $\mathcal{R}$ , creating a set  $\tau_i$  for each  $p_i \in \mathcal{P}$ , and setting its weight  $\omega(\tau_i) = r(p_i)$ . 6.  $\mathcal{T}^* \leftarrow \emptyset$ 7. while  $|B| \ge \varepsilon$ 8. select  $\tau_i \in T$  that maximizes  $b(\tau_i)/\omega(\tau_i)$ 9.  $\mathcal{B} \leftarrow \mathcal{B} - \mathcal{B}(\tau_i)$ 10.  $\mathcal{T}^* \leftarrow \mathcal{T}^* \cup \{\tau_i\}$ 11. Return the set of predicates  $\mathcal{P}^*$  corresponding to  $\mathcal{T}^*$ .

Figure 5.4: The algorithm for learning disjunctive blocking

mation ratio of  $2\sqrt{t \log \beta}$  (Peleg, 2000). This algorithm is particularly appropriate for the adaptive blocking setting as it involves early discarding of particularly costly sets (blocking predicates that cover too many non-coreferent pairs), leading to more space-efficient learning of the blocking function. In the remaining discussion, we use the term "blocking predicates" in place of "sets" considered in the original set cover problem.

The outline of the algorithm APPROXRBSETCOVER is shown in Figure 5.4. The algorithm is provided with training data in the form of  $\beta$  coreferent record pairs  $\mathcal{B} = \{b_1, \ldots, b_\beta\}$  and  $\rho$  non-coreferent records pairs  $\mathcal{R} = \{r_1, \ldots, r_\beta\}$ , where each  $r_i$  and  $b_i$  represents a record pair  $(x_{i_1}, x_{i_2})$ . For each predicate  $p_i \in \mathcal{P}$ , let *covered negatives*  $\mathcal{R}(p_i)$  be the set of negative pairs it covers, *predicate cost*  $r(p_i)$  be the number of negative pairs it covers, and

coverage  $b(p_i)$  be the number of covered positives,  $b(p_i) = |\mathcal{B}(p_i)|$ . For each negative pair  $r_i = (x_{i_1}, x_{i_2})$ , let the *degree* deg $(r_i, \mathcal{P})$  be the number of predicates in  $\mathcal{P}$  that cover it; degree for a positive pair, deg $(b_i, \mathcal{P})$ , is defined analogously. In step 1 of the algorithm, blocking predicates that cover too many negative pairs are discarded, where the parameter  $\eta$  can be set to a fraction of the total number of pairs in the dataset. Then, negative pairs covered by too many predicates are discarded in step 4, which intuitively corresponds to disregarding non-coreferent pairs that are highly similar and are placed in the same block by most predicates. Again, this parameter can be set as a fraction of the available predicate set.

Next, a standard weighted set cover problem is set up for the remaining predicates and pairs by setting the cost of each predicate to be the number of negatives it covers and removing the negatives. The resulting weighted set cover problem is solved in steps 6-11 using Chvatal's greedy approximation algorithm (Chvatal, 1979). The algorithm iteratively constructs the cover, at each step adding the blocking predicate  $p_i$  that maximizes a greedy heuristic: the ratio of the number of previously uncovered positives over the predicate cost. To soften the constraint requiring all positive pairs to be covered, we add an early stopping condition permitting up to  $\varepsilon$  positives to remain uncovered. In practice,  $\varepsilon$  should be set to 0 at first, and then gradually increased if the cover identified by the algorithm is too costly for the application at hand (that is, when covering all positives incurs covering too many negatives).

#### **DNF Blocking**

Learning DNF blocking can be viewed as an extension of learning disjunctive blocking where not only individual blocking predicates may be selected, but also their conjunctions. We assume that conjunctions that include up to k predicates are considered. Because enumerating over all possible conjunctions of predicates results in an exponential number of predicate sets under consideration, we propose a two-stage procedure, shown in Figure 5.5.

First, a set of t(k-1) predicate conjunctions of lengths from 2 to k is created in



Figure 5.5: The algorithm for learning DNF blocking

a greedy fashion. Candidate conjunctions are constructed iteratively starting with each predicate  $p_i \in \mathcal{P}$ . At each step, another predicate is added to the current conjunction so that the ratio between the number of positives and the number of negatives covered by the conjunction is maximally improved.

After the candidate set of conjunctions of lengths from 2 to k is constructed, the conjunctions are added to  $\mathcal{P}$ , the set of individual predicates. Then, the APPROXRBSET-COVER algorithm described in the previous section is used to learn a blocking function that corresponds to a DNF formula over the blocking predicates.

#### **5.3.3 Blocking with the Learned Functions**

Efficiency considerations, which are the primary motivation for this work, require the learned blocking functions to perform the actual blocking on new, unlabeled data in an effective manner. After the blocking function is learned using training data, it should be applied to the test data (for the actual linkage or clustering task) without explicitly constructing all pairs of records and evaluating the predicates on them. This is achieved by applying the indexing function for every blocking predicate or conjunction in the learned blocking function to every record in the test dataset. Thus, an inverted index is constructed for each predicate or conjunction in the blocking function. In each inverted index, every key is associated with a list of instances for which the indexing function of the corresponding predicate returns the key value. Disjunctive and DNF blocking can then be performed by iterating over every key in all inverted indices and returning all pairs of records that occur in the same list for any key.

### **5.4 Experimental Results**

#### 5.4.1 Methodology and Datasets

We evaluate the efficiency of the our methods for learning blocking functions using two metrics, speedup ratio and recall. They are are defined with respect to the number of coreferent and non-coreferent record pairs that get covered by a blocking function  $f_{\mathcal{P}}$  in a database of *n* records:

$$\begin{aligned} \text{ReductionRatio} &= 1.0 - \frac{\sum_{(x_i, x_j) \in \mathcal{R}} f_{\mathcal{P}}(x_i, x_j) + \sum_{(x_i, x_j) \in \mathcal{B}} f_{\mathcal{P}}(x_i, x_j)}{n(n-1)/2} \\ \text{Recall} &= \frac{\sum_{(x_i, x_j) \in \mathcal{B}} f_{\mathcal{P}}(x_i, x_j)}{|\mathcal{B}|} \end{aligned}$$

Intuitively, recall captures blocking accuracy by measuring the proportion of truly coreferent record pairs that have been covered by the blocking function. an ideal blocking function would have recall of 1.0, indicating that all coreferent pairs are covered. Reduction ratio measures the efficiency gain due to blocking by measuring what proportion of all pairs in the dataset is filtered out by the blocking function. Without blocking, reduction ratio is 0

since all record pairs are returned, while a higher number indicates what proportion of pairs is not covered, and therefore will not require similarity computations in the subsequent record linkage stages or in the clustering algorithm. Note that efficiency savings due to blocking are more substantial if collective (graph-based) inference methods are used for linkage or clustering (Pasula et al., 2003; McCallum & Wellner, 2004a; Singla & Domingos, 2005; Bhattacharya & Getoor, 2006), as the time complexity of these methods increases superlinearly with the number of record pairs under consideration.

Results are obtained using 10 runs of two-fold cross-validation. Using a higher number of folds would result in fewer coreferent records in the test fold, which would artificially make the blocking task easier. During each run, the dataset is split into two folds by randomly assigning all records for every underlying entity to one of the folds. The blocking function is then trained using record pairs generated from the training fold. The learned blocking function is used to perform blocking on the test fold, based on which recall and reduction ratio are measured.

We present results on two datasets: *Cora* and *Addresses*. The *Cora* dataset is described in Section 3.1.1. While it is a relatively small-scale dataset, results of Chapter 3 illustrate that good linkage performance on this domain requires computationally intensive string similarity functions; it has also been shown that linkage on that dataset benefits from collective linkage methods (Singla & Domingos, 2005), justifying the need for blocking. *Addresses* is a dataset containing names and addresses of 50,000 9-field records for 10,000 unique individuals that was generated using the DBGEN program provided by Hernández and Stolfo (1995).

We use the following general predicates are for constructing learnable blocking functions:

- *Exact Match*: covers instances that have the same value for the field;
- *Contain Common Token*: covers instances that contain a common token in the field value;

- *Contain Common Integer*: covers instances that contain a common token consisting of digits in the field value;
- *Contain Same or Off-by-One Integer*: covers instances that contain integer tokens that are equal or differ by at most 1;
- *Same N First Chars, N* = 3, 5, 7: covers instances that have a common character prefix in the field value;
- *Contain Common Token N-gram, N* = 2, 4, 6: covers instances that contain a common length-*N* subsequence of tokens;
- *Token-based TF-IDF* >  $\delta$ ,  $\delta = 0.2, 0.4, 0.6, 0.8, 1.0$ : covers instances where tokenbased TF-IDF cosine similarity between field values is greater than the threshold  $\delta$ ;
- *N-gram-based TF-IDF* >  $\delta$ ,  $\delta = 0.2, 0.4, 0.6, 0.8, 1.0$ , N = 3, 5: covers instances where TF-IDF cosine similarity between n-gram representations of field values is greater than the threshold  $\delta$ .

As described in Section 5.2, these general predicates are instantiated for all fields in the given database. Algorithms presented in Section 5.3.2 are used to construct blocking functions by selecting subsets of the resulting field-specific predicates. For DNF blocking, conjunctions of length 2 were employed, as experiments with longer conjunctions did not lead to improvements over blocking based on a 2-DNF.

We vary the value of parameter  $\varepsilon$  (which specifies the number of coreferent pairs allowed to remain uncovered) by setting to  $r\beta$  for different values of desired recall r between 0.0 and 1.0, where  $\beta$  is the number of coreferent record pairs in the training fold. This parameter captures the dependence between the reduction ratio and recall: if  $\varepsilon$  is high, fewer predicates are selected resulting in lower recall since not all coreferent pairs are retrieved. At the same time, the reduction ratio is higher for higher  $\varepsilon$  since fewer pairs are covered by the learned blocking function, leading to higher efficiency. By varying  $\varepsilon$ , we obtain a series of results that demonstrate the trade-off between obtaining higher recall and improving the reduction ratio.

We compare the proposed methods with CANOPIES (McCallum et al., 2000), a popular blocking method relying on token-based or n-gram-based TF-IDF similarity computed using an inverted index. In a previous study, Baxter *et al.* (Baxter et al., 2003) have compared several manually-tuned blocking strategies and found CANOPIES to produce best overall results. CANOPIES also allows trading off precision and the reduction ratio by varying the threshold parameter that controls the coverage of the blocking.<sup>1</sup> We tried both token-based CANOPIES and tri-gram based CANOPIES and chose the best-performing variants as baselines: token-based indexing for *Cora*, and tri-gram indexing for *Addresses*. This difference is due to the fact that most variation between coreferent citations in *Cora* is due to insertions and deletions of whole words, making token-based similarity more appropriate. Coreferent records in *Addresses*, on other hand, mostly differ due to misspellings and character-level transformations that n-gram similarity is able to capture.

#### 5.4.2 Results and Discussion

Figures 5.6 and 5.7 show the reduction ratio versus recall curves for the two types of learned blocking functions described above and for CANOPIES. From these results, we observe that both variants of adaptive blocking outperform the unlearned baseline: combining multiple predicates allows achieving higher recall levels as well as achieving higher reduction ratios. DNF blocking is more accurate than disjunctive blocking, and on *Addresses* it also achieves higher recall, while for *Cora* the maximum recall is comparable. Because DNF blocking is based on predicate conjunctions, non-coreferent pairs are easier avoided by the blocking function: conjunctions effectively form high-precision, low-recall rules that cover smaller subsets of coreferent pairs but fewer non-coreferent pairs compared to single predicates.

<sup>&</sup>lt;sup>1</sup>The original CANOPIES algorithm allows varying two separate threshold parameters, however, empirical results have shown that using the same value for both thresholds yields highest performance (McCallum et al., 2000).



Figure 5.6: Blocking accuracy results for the Cora dataset



Figure 5.7: Blocking accuracy results for the Addresses dataset

While none of the methods achieve 100% recall (as it would effectively require returning all record pairs), for both datasets adaptive blocking is able to achieve higher recall than CANOPIES. Thus, using learnable blocking functions leads to *both* accuracy and efficiency improvements.

	Cora	Addresses
DNF Blocking,	23,499	4,890,410
Disjunctive Blocking	41,439	4,090,283
Canopies	125,986	1,745,995
Total number of pairs	606,182	312,487,500

Table 5.1: Average number of pairs covered by the learned blocking functions and highest achieved recall

Table 5.1 shows the actual number of record pairs returned by the different blocking methods at highest achieved recall. These results demonstrate the significance of differences in the reduction ratio between the different blocking functions: because the total number of pairs is very large, differences in the reduction ratio translate into significant savings in the number of pairs for which similarity must be computed. Note that the smaller number of pairs returned by CANOPIES and disjunctive blocking on *Addresses* corresponds to a significantly lower recall, while for a fixed recall level DNF blocking either does as well or better.

	Cora	Addresses
DNF Blocking	26.9	735.81
Disjunctive Blocking	32.4	409.4
Canopies	16.0	572.7

Table 5.2: Average blocking time, CPU seconds

Table 5.2 show the blocking times for the different methods measured at maximum achieved recall. Learnable blocking functions incur a relatively modest increase in computational time despite the fact that they utilize many predicates. This is due to the fact that the learned predicates that cover few negatives typically require smaller inverted indices

than the one built by canopies using tokens or n-grams where each token or n-gram occurs in many strings. Many predicates employed by the adaptive blocking functions, on other hand, map each string to a single key, resulting in more efficient retrieval of covered pairs. Inverted indices corresponding to conjunctions are even more efficient as they contain many keys (the cross product of the key sets for the predicates in the conjunction) and incur less chaining, which is the reason for better performance of DNF blocking compared to disjunctive blocking on *Cora*, where the number of predicates in the constructed blocking function is similar for the two methods. On *Addresses*, DNF blocking constructs blocking functions containing more predicates, which on one hand incurs a computational penalty, but on other allows it to achieve higher recall.

Overall, the results demonstrate that adaptive blocking functions significantly improve the efficiency of record linkage, and provide an attractive methodology for scaling up data mining tasks that rely on similarity computations between pairs of instances.

## 5.5 Related Work

A number of blocking methods have been proposed by researchers for speeding up record linkage and clustering (Fellegi & Sunter, 1969; Kelley, 1985; Newcombe, 1988; Jaro, 1989; Hernández & Stolfo, 1995; McCallum et al., 2000; Baxter et al., 2003; Chaudhuri et al., 2003; Jin et al., 2003; Gu & Baxter, 2004; Winkler, 2005); see the summary of these methods in Section 2.4. A key distinction between prior work and our approach is that previously described methods focus on improving blocking efficiency while assuming that an accurate blocking function is known and its parameters have been tuned manually. In contrast, our approach attempts to construct an optimal blocking function automatically. Because blocking functions can be learned using any combination of similarity predicates on different record fields, and no assumptions are made about the number of record fields or their type, our approach can be used for adapting the blocking function in any domain, while allowing human experts to add domain-specific predicates.

Our predicate-based blocking approach is most closely related to key-based methods developed by researchers working on record linkage for Census data (Kelley, 1985; Newcombe, 1988; Jaro, 1989; Winkler, 2005). Techniques described by Kelley (Kelley, 1985) and Winkler (Winkler, 2005) are particularly relevant as they describe methodologies for evaluating the accuracy of individual blocking predicates, and could be integrated with our approach for further speedup of blocking function learning.

Our formulation for training disjunctive and DNF blocking functions is related to machine learning algorithms for learning disjunctive rules and DNFs (Mitchell, 1997). A principal difference between that work and the learnable blocking problem is that in our setting the learned disjunctions must cover all positive record pairs while minimizing the number of covered negative pairs, while rule learning methods generally attempt to equally minimize the number of errors on both positive and negative examples. Cost-sensitive machine learning methods (Elkan, 2001) may provide a foundation for other approaches to adaptive blocking, and we hope that our initial work will encourage the development of alternative learnable blocking techniques.

### 5.6 Chapter Summary

In this chapter, we formulated the adaptive blocking problem as the task of learning a function that returns a minimal number of non-coreferent record pairs while returning all or nearly all coreferent pairs. We described two types of blocking functions: disjunctive and DNF blocking. Formulating the learning problem as an instance of the Red-Blue Set Cover problem allowed us to adopt a well-known approximation algorithm for that problem to construct blocking functions. Experimental results demonstrated the ability of our approach to learn efficient and accurate blocking functions automatically.

# **Chapter 6**

# **Future Work**

Because learnable similarity functions are a part of many machine learning and data analysis tasks, there is a large number of applications where adapting distance computations can have a significant effect on performance. These applications can be found in such fields as natural language processing, information retrieval, vision, robotics and bioinformatics, where application-specific similarity functions are often employed. Adapting such functions *in situ* in these applications can be achieved using the framework used in the three applications considered in this thesis: learning from pairwise supervision. While specific applications in the above areas are beyond the scope of this thesis, in subsequent sections we describe several directions for future work that are directly related to the applications and similarity functions considered in prior chapters.

# 6.1 Multi-level String Similarity Functions

Improvements obtained using learnable affine-gap edit distance over its unlearned equivalent demonstrated the benefits of adapting string similarity computations to a given domain. However, edit distance has certain properties that may limit its suitability in some domains. For example, it does not directly handle transpositions of entire fragments (e.g., token swaps), and while edit operations for short-range transpositions can be added at considerable computational cost, handling long-term transpositions is problematic. Orderinsensitive similarity functions such as cosine similarity, on other hand, have no trouble dealing with token transpositions, yet they depend on accurate tokenization and suffer when edit operations occur at the character level.

The SoftTFIDF variant of cosine similarity recently proposed by Cohen et al. (2003a) attempts to amend this drawback of cosine similarity, yet it cannot adapt to a given domain beyond the IDF weighting. An exciting challenge for future work lies in developing learnable string similarity functions that integrate adaptive string comparison at the character, token, and document (string) levels. Such similarity functions must rely on joint similarity computation across the levels while remaining computationally efficient. While segmented pair HMMs presented in Section 3.1.2 are a first step in this direction, developing string similarity models that perform further structural analysis of strings remains an open research issue. Progress in this area will have impact in all tasks that rely on string similarity functions such as record linkage and information retrieval.

## 6.2 Discriminative Pair HMMs

The Expectation-Maximization algorithm that we described in Chapter 3 for training pair HMMs only utilizes positive supervision: the learning procedure maximizes the likelihood of observing alignments of coreferent pairs. However, it may be advantageous to exploit negative supervision, that is, pairs of non-coreferent strings, since some "near-miss" negative examples can be very informative.

A recently proposed edit distance model based on Conditional Random Fields (CRFs) has structure that allows training with both positive and negative examples so that the model directly learns to discriminate between the two kinds of pairs (McCallum et al., 2005). The CRF edit distance model consists of two three-state edit distance transducers, one of which computes the alignment probabilities for coreferent strings, while the other computes align-

ment probabilities for non-coreferent strings.

Although the CRF-based model has different probabilistic semantics (alignments are not generated since the model is conditioned on the fact that any two strings under consideration are aligned), the coupled structure of that model can be implemented as a pair HMM. Considering such coupled structures within the pair HMM framework is an interesting area for future work, since it would allow applying discriminative training methods that explicitly attempt to learn model parameters that effectively distinguish between coreferent and non-coreferent strings (Eisner, 2002). Another avenue for future work on alternative pair HMM structures involves deriving learnable models for *local* alignment that focus on scoring matching alignment fragments while disregarding the mismatched sequences around them (Gusfield, 1997). In domains where large gaps are commonplace yet small matching sequences may be very informative, e.g., in linkage of retail product descriptions, pair HMM structures that model local alignment may yield better performance, and investigating this possibility is an interesting future direction.

### 6.3 Active Learning of Similarity Functions

As discussed in Section 3.3, the goal of active learning methods for similarity functions is identifying pairs of objects whose equivalence or non-equivalence is informative for improving distance estimates. The classifier-based record similarity described in Section 3.2 lends itself nicely to active learning techniques developed for classification, which has been explored by Sarawagi and Bhamidipaty (2002) and Tejada et al. (2002) in the record linkage context.

One of the biggest challenges in selecting useful training example pairs lies with the fact that the space of possible pairs grows quadratically with the number of examples, and static-active and weakly-labeled methodologies we proposed in Section 3.3 address this challenge. However, these methods are based on heuristics, while developing more principled active learning methods remains an interesting direction for future work. Such methods must directly attempt to identify example pairs that would lead to maximal improvement of similarity estimates. Traditional active learning approaches such as uncertainty sampling (Lewis & Catlett, 1994), query-by-committee (Seung et al., 1992), estimation error reduction (Lindenbaum et al., 1999; Roy & McCallum, 2001), and version space reduction (Tong, 2001) could be adopted for this task, and developing such methods for directly improving the learning of similarity functions like edit distance or distortion measures described in Chapter 4 is an area yet to be explored.

### 6.4 From Adaptive Blocking to Learnable Metric Mapping

The predicate-based methodology that we proposed in Chapter 5 for automatically obtaining accurate blocking functions requires specifying an initial set of blocking predicates. Although a sufficiently general set of predicates for textual data is easy to encode, in future work it would be interesting to explore learnable blocking methods that are not predicatebased but rely on mapping records to metric spaces. Several existing blocking methods rely on such mapping, such as those of Jin et al. (2003) and Chaudhuri et al. (2003). Learning algorithms that would make these methods adaptive could pursue two directions: searching for an optimal mapping of data to metric space, or transforming the metric space after the mapping to allow efficient yet accurate selection of approximately similar records.

This problem is related to methods for fast nearest-neighbor searching, a number of which have been developed in the past decade (Indyk & Motwani, 1998; Liu, Moore, Gray, & Yang, 2004; Beygelzimer, Kakade, & Langford, 2006). However, using these techniques for domains where data is described by multiple fields of heterogeneous types is non-trivial as they typically rely on strong metric assumptions on the data space, and do not scale efficiently to high-dimensional data. Developing adaptive nearest-neighbor search methods for heterogeneous data is an interesting area for future work that has applications in blocking as well as in other tasks where retrieving approximately similar objects efficiently is important, e.g., in classification methods and database retrieval.

# **Chapter 7**

# Conclusions

Research presented in this thesis has focused on learning similarity functions from pairwise supervision. We have shown that by parameterizing several popular distance functions and learning parameter values from examples of similar and dissimilar instance pairs, we obtain increases in accuracy of similarity computations, which lead to performance improvements in tasks that rely on them: record linkage, semi-supervised clustering, and blocking.

First, we have considered learning similarity functions in the context of record linkage where they are used for two tasks: computing similarity between individual field values and combining these similarities across multiple fields. For field-level similarity computations, we have described two adaptive variants of affine-gap edit distance in which the costs of string transformations are learned on a corpus of coreferent string pairs. Our approach is based on pair HMMs, a probabilistic model for generating string alignments. Learning the costs of affine-gap edit distance parameters allows adapting the underlying string matching algorithm to each field's domain, while using segmented pair HMMs integrates such adaptation with performing string segmentation that is helpful in domains where strings are composed of multiple fields from different domains.

For computing similarity between records in linkage, we have demonstrated that Support Vector Machines (SVMs) effectively combine similarities from individual fields in proportion to their relative importance. Using learnable similarity functions at both field and record levels leads to improved results over using record-level learnable similarity functions that combine unlearned field-level similarities.

We have proposed two strategies for selecting informative pairs of coreferent or non-coreferent examples for training similarity functions in record linkage. One of the proposed strategies, weakly-labeled negative selection does not require labeled supervision, while the other, likely positive pair selection, avoids the computational costs of the standard active learning methods. Both of these strategies facilitate efficient selection of training pairs that allows learning accurate similarity functions on small training sets.

Second, we have demonstrated the utility of employing learnable similarity functions in semi-supervised clustering. By incorporating similarity function learning within the HMRF-KMEANS algorithm for semi-supervised clustering, we were able to leverage both labeled pairwise supervision and unlabeled data when adapting the similarity functions. Our approach allows learning individual similarity functions for different clusters which is useful for domains where clusters have different shapes. The proposed framework can be used with a variety of distortion (distance) functions that include directional measures such as cosine similarity, and Bregman divergences that include Euclidean distance and Kullback Leibler divergence. Ablation experiments have demonstrated that the HMRF-based approach combines the strengths of learnable similarity functions and constrained clustering to obtain significant improvements in clustering quality.

In the context of blocking, the third application we considered, we have proposed methods for learning similarity functions that efficiently select approximately similar pairs of examples. Because blocking is required for scaling record linkage and many pairwise clustering algorithms up to large datasets, our technique shows that learnable similarity functions can be employed not only for increasing accuracy of data mining tasks, but also for improving their scalability. Unlike previous blocking methods that require manual tuning and hand-construction of blocking functions, our approach is adaptive as it optimizes the blocking function for a given domain using pairwise supervision that can be naturally obtained in linkage and clustering tasks.

For the three tasks under consideration, we have evaluated the effectiveness of utilizing learnable similarity functions, comparing their accuracy on standard benchmarks with that of unlearned similarity functions typically used in these tasks. Our experiments demonstrate that learnable similarity functions effectively utilize the pairwise training data to make distance estimates more accurate for a given domain, resulting in overall performance improvements on the tasks.

Overall, the work presented in this thesis contributes methods leading to state-ofthe art performance on the considered tasks and provides a number of useful algorithms for practitioners in record linkage, semi-supervised clustering, and blocking. This research demonstrates the power of using similarity functions that can adapt to a given domain using pairwise supervision, and we hope that it will motivate further research in trainable distance functions, as well as encourage employing such functions in various applications where distance estimates between instances are required.

# **Bibliography**

- Aha, D. W. (1998). Feature weighting for lazy learning algorithms. In Liu, H., & Motoda,
  H. (Eds.), *Feature Extraction, Construction and Selection: A Data Mining Perspective*. Kluwer Academic Publishers, Norwell, MA.
- Ananthakrishna, R., Chaudhuri, S., & Ganti, V. (2002). Eliminating fuzzy duplicates in data warehouses. In *Proceedings of the 28th International Conference on Very Large Databases (VLDB-2002)* Hong Kong, China.
- Baeza-Yates, R., & Ribeiro-Neto, B. (1999). Modern Information Retrieval. ACM Press, New York.
- Banerjee, A., Dhillon, I., Ghosh, J., & Sra, S. (2005a). Clustering on the unit hypersphere using von Mises-Fisher distributions. *Journal of Machine Learning Research*, 6, 1345–1382.
- Banerjee, A., Merugu, S., Dhilon, I., & Ghosh, J. (2005b). Clustering with Bregman divergences. *Journal of Machine Learning Research*, 6, 1705–1749.
- Banerjee, A., Krumpelman, C., Basu, S., Mooney, R. J., & Ghosh, J. (2005c). Model-based overlapping clustering. In Proceedings of the Eleventh ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD-05).
- Bar-Hillel, A., Hertz, T., Shental, N., & Weinshall, D. (2003). Learning distance functions

using equivalence relations. In *Proceedings of 20th International Conference on Machine Learning (ICML-2003)*, pp. 11–18.

- Basu, S. (2005). Semi-supervised Clustering: Probabilistic Models, Algorithms and Experiments. Ph.D. thesis, University of Texas at Austin.
- Basu, S., Banerjee, A., & Mooney, R. J. (2002). Semi-supervised clustering by seeding. In Proceedings of 19th International Conference on Machine Learning (ICML-2002), pp. 19–26.
- Basu, S., Banerjee, A., & Mooney, R. J. (2004). Active semi-supervision for pairwise constrained clustering. In *Proceedings of the 2004 SIAM International Conference* on Data Mining (SDM-04).
- Basu, S., Bilenko, M., Banerjee, A., & Mooney, R. J. (2006). Probabilistic semi-supervised clustering with constraints. In Chapelle, O., Schölkopf, B., & Zien, A. (Eds.), Semi-Supervised Learning. MIT Press, Cambridge, MA.
- Basu, S., Bilenko, M., & Mooney, R. J. (2004). A probabilistic framework for semisupervised clustering. In *Proceedings of the Tenth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD-2004)*, pp. 59–68 Seattle, WA.
- Baxter, R., Christen, P., & Churches, T. (2003). A comparison of fast blocking methods for record linkage. In *Proceedings of the 2003 ACM SIGKDD Workshop on Data Cleaning, Record Linkage, and Object Consolidation*, pp. 25–27 Washington, DC.
- Beygelzimer, A., Kakade, S., & Langford, J. (2006). Cover trees for nearest neighbor. In Proceedings of 23rd International Conference on Machine Learning (ICML-2006).
- Bhattacharya, I., & Getoor, L. (2004). Deduplication and group detection using links. In Proceedings of the 2004 ACM SIGKDD Workshop on Link Analysis and Group Detection.

- Bhattacharya, I., & Getoor, L. (2006). A latent dirichlet model for unsupervised entity resolution. In *6th SIAM Conference on Data Mining (SDM-2006)* Bethesda, MD.
- Bilenko, M., Kamath, B., & Mooney, R. J. (2006). Adaptive blocking: Learning to scale up record linkage. In Proceedings of the WWW-2006 Workshop on Information Integration on the Web.
- Bilenko, M., & Basu, S. (2004). A comparison of inference techniques for semi-supervised clustering with hidden Markov random fields. In *Proceedings of the ICML-2004 Workshop on Statistical Relational Learning and its Connections to Other Fields (SRL-2004)* Banff, Canada.
- Bilenko, M., Basu, S., & Mooney, R. J. (2004). Integrating constraints and metric learning in semi-supervised clustering. In *Proceedings of 21st International Conference on Machine Learning (ICML-2004)*, pp. 81–88 Banff, Canada.
- Bilenko, M., Basu, S., & Sahami, M. (2005). Adaptive product normalization: Using online learning for record linkage in comparison shopping. In *Proceedings of the 5th IEEE International Conference on Data Mining (ICDM-2005)*, pp. 58–65.
- Bilenko, M., & Mooney, R. J. (2002). Learning to combine trained distance metrics for duplicate detection in databases. Tech. rep. AI 02-296, Artificial Intelligence Laboratory, University of Texas at Austin, Austin, TX.
- Bilenko, M., & Mooney, R. J. (2003a). Adaptive duplicate detection using learnable string similarity measures. In *Proceedings of the Ninth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD-2003)*, pp. 39–48 Washington, DC.
- Bilenko, M., & Mooney, R. J. (2003b). On evaluation and training-set construction for duplicate detection. In *Proceedings of the KDD-03 Workshop on Data Cleaning*, *Record Linkage, and Object Consolidation*, pp. 7–12 Washington, DC.

- Blake, C. L., & Merz, C. J. (1998). UCI repository of machine learning databases. http://www.ics.uci.edu/~mlearn/MLRepository.html.
- Buntine, W. L. (1994). Operations for learning graphical models. Journal of Artificial Intelligence Research, 2, 159–225.
- Carr, R. D., Doddi, S., Konjevod, G., & Marathe, M. (2000). On the Red-Blue Set Cover problem. In Proceedings of the 11th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA-2000) San Francisco, CA.
- Chaudhuri, S., Ganjam, K., Ganti, V., & Motwani, R. (2003). Robust and efficient fuzzy match for online data cleaning. In *Proceedings of the 2003 ACM SIGMOD International Conference on Management of Data (SIGMOD-03)*, pp. 313–324. ACM Press.
- Chvatal, V. (1979). A greedy heuristic for the set covering problem. *Mathematics of Operations Research*, 4(3), 233–235.
- Cohen, W. W. (1998). Integration of heterogeneous databases without common domains using queries based on textual similarity. In *Proceedings of the 1998 ACM SIGMOD International Conference on Management of Data (SIGMOD-98)*, pp. 201–212 Seattle, WA.
- Cohen, W. W., Kautz, H., & McAllester, D. (2000). Hardening soft information sources. In Proceedings of the Sixth International Conference on Knowledge Discovery and Data Mining (KDD-2000) Boston, MA.
- Cohen, W. W., Ravikumar, P., & Fienberg, S. E. (2003a). A comparison of string distance metrics for name-matching tasks. In *Proceedings of the IJCAI-2003 Workshop on Information Integration on the Web*, pp. 73–78 Acapulco, Mexico.
- Cohen, W. W., Ravikumar, P., & Fienberg, S. E. (2003b). A comparison of string metrics for matching names and records. In *Proceedings of the 2003 ACM SIGKDD Workshop on*

Data Cleaning, Record Linkage, and Object Consolidation, pp. 13–18 Washington, DC.

- Cohen, W. W., & Richman, J. (2002). Learning to match and cluster large high-dimensional data sets for data integration. In *Proceedings of the Eighth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD-2002)*, pp. 475–480 Edmonton, Alberta.
- Cohn, D., Caruana, R., & McCallum, A. (2003). Semi-supervised clustering with user feedback. Tech. rep. TR2003-1892, Cornell University.
- Cohn, D. A., Ghahramani, Z., & Jordan, M. I. (1996). Active learning with statistical models. *Journal of Artificial Intelligence Research*, 4, 129–145.
- Cooper, G. G., & Herskovits, E. (1992). A Bayesian method for the induction of probabilistic networks from data. *Machine Learning*, *9*, 309–347.
- Cover, T. M., & Thomas, J. A. (1991). Elements of Information Theory. Wiley-Interscience.
- Dempster, A. P., Laird, N. M., & Rubin, D. B. (1977). Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society B*, *39*, 1–38.
- Dhillon, I. S., & Modha, D. S. (2001). Concept decompositions for large sparse text data using clustering. *Machine Learning*, 42, 143–175.
- Dhillon, I. S., & Guan, Y. (2003). Information theoretic clustering of sparse co-occurrence data. In *Proceedings of the Third IEEE International Conference on Data Mining* (*ICDM-2003*), pp. 517–521.
- Dom, B. E. (2001). An information-theoretic external cluster-validity measure. Research report RJ 10219, IBM.

- Dong, X., Halevy, A., & Madhavan, J. (2005). Reference reconciliation in complex information spaces. In Proceedings of the 2005 ACM SIGMOD international conference on Management of data (SIGMOD-2005), pp. 85–96 Baltimore, MD.
- Duda, R. O., Hart, P. E., & Stork, D. G. (2001). Pattern Classification (Second edition). Wiley, New York.
- Durbin, R., Eddy, S., Krogh, A., & Mitchison, G. (1998). Biological Sequence Analysis: Probabilistic Models of Proteins and Nucleic Acids. Cambridge University Press.
- Eisner, J. (2002). Parameter estimation for probabilistic finite-state transducers. In *Proceed*ings of the 40th Annual Meeting of the Association for Computational Linguistics (ACL-2002).
- Elfeky, M. G., Elmagarmid, A. K., & Verykios, V. S. (2002). TAILOR: A record linkage tool box. In *Proceedings of the 18th International Conference on Data Engineering* (*ICDE-2002*), pp. 17–28.
- Elkan, C. (2001). The foundations of cost-sensitive learning. In *Proceedings of the Seven*teenth International Joint Conference on Artificial Intelligence (IJCAI-2001).
- Fellegi, I. P., & Sunter, A. B. (1969). A theory for record linkage. *Journal of the American Statistical Association*, 64(328), 1183–1210.
- Fern, X., & Brodley, C. (2003). Random projection for high dimensional data clustering: A cluster ensemble approach. In *Proceedings of 20th International Conference on Machine Learning (ICML-2003)*.
- Freitag, D., & McCallum, A. (1999). Information extraction with HMMs and shrinkage. In Papers from the Sixteenth National Conference on Artificial Intelligence (AAAI-99)
   Workshop on Machine Learning for Information Extraction, pp. 31–36 Orlando, FL.

- Freund, Y., & Schapire, R. E. (1996). Experiments with a new boosting algorithm. In Proceedings of the Thirteenth International Conference on Machine Learning (ICML-96), pp. 148–156 Bari, Italy. Morgan Kaufmann.
- Friedman, J. H. (1997). On bias, variance, 0/1-loss, and the curse-of-dimensionality. Data Mining and Knowledge Discovery, 1(1), 55–77.
- Geman, S., & Geman, D. (1984). Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 6, 721–742.
- Giles, C. L., Bollacker, K., & Lawrence, S. (1998). CiteSeer: An automatic citation indexing system. In *Proceedings of the Third ACM Conference on Digital Libraries*, pp. 89–98 Pittsburgh, PA.
- Golub, G. H., & van Loan, C. F. (1989). *Matrix Computations* (Second edition). Johns Hopkins University Press.
- Gotoh, O. (1982). An improved algorithm for matching biological sequences. Journal of Molecular Biology, 162, 705–708.
- Grenager, T., Klein, D., & Manning, C. D. (2005). Unsupervised learning of field segmentation models for information extraction. In *Proceedings of the 43nd Annual Meeting* of the Association for Computational Linguistics (ACL-05).
- Gu, L., & Baxter, R. (2004). Adaptive filtering for efficient record linkage. In Proceedings of the Fourth SIAM International Conference on Data Mining (SDM-04).
- Gusfield, D. (1997). *Algorithms on Strings, Trees and Sequences*. Cambridge University Press, New York.
- Hammersley, J., & Clifford, P. (1971). Markov fields on graphs and lattices. Unpublished manuscript.

- Hernández, M. A., & Stolfo, S. J. (1995). The merge/purge problem for large databases. In Proceedings of the 1995 ACM SIGMOD International Conference on Management of Data (SIGMOD-95), pp. 127–138 San Jose, CA.
- Hofmann, T., & Buhmann, J. M. (1998). Active data clustering. In Advances in Neural Information Processing Systems 10.
- Indyk, P., & Motwani, R. (1998). Approximate nearest neighbors: towards removing the curse of dimensionality. In *Proceedings of the 30th ACM Symposium on Theory of Computing (STOC-98)*, pp. 604–613.
- Jain, A. K., Murty, M. N., & Flynn, P. J. (1999). Data clustering: A review. ACM Computing Surveys, 31(3), 264–323.
- Jaro, M. (1989). Advances in record-linkage methodology as applied to matching the 1985 census of Tampa, Florida. *Journal of the American Statistical Association*, 84, 414– 420.
- Jelinek, F., & Mercer, R. (1980). Interpolated estimation of markov source parameters from sparse data. In *Pattern Recognition in Practice*, pp. 381–402.
- Jelinek, F. (1998). Statistical Methods for Speech Recognition. MIT Press, Cambridge, MA.
- Jin, L., Li, C., & Mehrotra, S. (2003). Efficient record linkage in large data sets. In Proceedings of the 8th International Conference on Database Systems for Advanced Applications (DASFAA-03), pp. 137–148 Kyoto, Japan.
- Joachims, T. (2003). Learning to align sequences: A maximum-margin approach. Tech. rep., Cornell University, Department of Computer Science.
- Karypis, G., & Kumar, V. (1998). A fast and high quality multilevel scheme for partitioning irregular graphs. SIAM Journal on Scientific Computing, 20(1), 359–392.

- Kaufman, L., & Rousseeuw, P. (1990). Finding Groups in Data: An Introduction to Cluster Analysis. John Wiley and Sons, New York.
- Kearns, M., Mansour, Y., & Ng, A. Y. (1997). An information-theoretic analysis of hard and soft assignment methods for clustering. In *Proceedings of 13th Conference on Uncertainty in Artificial Intelligence (UAI-97)*, pp. 282–293.
- Kelley, R. P. (1985). Advances in record linkage methodology: a method for determining the best blocking strategy. In *Record Linkage Techniques - 1985: Proceedings of the Workshop on Exact Matching Methodologies*, pp. 199–203 Arlington, VA.
- Klein, D., Kamvar, S. D., & Manning, C. (2002). From instance-level constraints to spacelevel constraints: Making the most of prior knowledge in data clustering. In *Proceedings of 19th International Conference on Machine Learning (ICML-2002)*, pp. 307–314 Sydney, Australia.
- Kschischang, F. R., Frey, B., & Loeliger, H.-A. (2001). Factor graphs and the sum-product algorithm. *IEEE Transactions on Information Theory*, 47(2), 498–519.
- le Cessie, S., & van Houwelingen, J. C. (1992). Ridge estimators in logistic regression. *Applied Statistics*, 41(1), 191–201.
- Levenshtein, V. I. (1966). Binary codes capable of correcting insertions and reversals. *Soviet Physics Doklady*, *10*(8), 707–710.
- Lewis, D. D., & Catlett, J. (1994). Heterogeneous uncertainty sampling for supervised learning. In Proceedings of the Eleventh International Conference on Machine Learning (ICML-94), pp. 148–156 San Francisco, CA. Morgan Kaufmann.
- Li, X., Morie, P., & Roth, D. (2004). Identification and tracing of ambiguous names: Discriminative and generative approaches. In *Proceedings of the 19th National Conference on Artificial Intelligence (AAAI-2004).*
- Lin, J. (1991). Divergence measures based on the shannon entropy. *IEEE Transactions on Information Theory*, 37(1), 145–151.
- Lindenbaum, M., Markovitch, S., & Rusakov, D. (1999). Selective sampling for nearest neighbor classifiers. In *Proceedings of the Sixteenth National Conference on Artificial Intelligence (AAAI-99)*, pp. 366–371.
- Liu, T., Moore, A., Gray, A., & Yang, K. (2004). An investigation of practical approximate nearest neighbor algorithms. In Advances in Neural Information Processing Systems 16.
- MacQueen, J. (1967). Some methods for classification and analysis of multivariate observations. In *Proceedings of 5th Berkeley Symposium on Mathematical Statistics and Probability*, pp. 281–297.
- McCallum, A., Bellare, K., & Pereira, F. (2005). A conditional random field for discriminatively-trained finite-state string edit distance. In *Proceedings of the 21st Conference on Uncertainty in Artificial Intelligence (UAI-2005).*
- McCallum, A., Nigam, K., & Ungar, L. (2000). Efficient clustering of high-dimensional data sets with application to reference matching. In *Proceedings of the Sixth International Conference on Knowledge Discovery and Data Mining (KDD-2000)*, pp. 169–178 Boston, MA.
- McCallum, A., & Wellner, B. (2004a). Conditional models of identity uncertainty with application to noun coreference. In Advances in Neural Information Processing Systems 17.
- McCallum, A., & Wellner, B. (2004b). Conditional models of identity uncertainty with application to noun coreference. In Advances in Neural Information Processing Systems 17.

- Meila, M. (2003). Comparing clusterings by the variation of information. In *Proceedings* of the 16th Annual Conference on Computational Learning Theory, pp. 173–187.
- Michalowski, M., Thakkar, S., & Knoblock, C. A. (2003). Exploiting secondary sources for automatic object consolidation. In *Proceedings of the ACM SIGKDD-03 Workshop on Data Cleaning, Record Linkage, and Object Consolidation*, pp. 34–36 Washington, DC.
- Minton, S. N., Nanjo, C., Knoblock, C. A., Michalowski, M., & Michelson, M. (2005). A heterogeneous field matching method for record linkage. In *Proceedings of the 5th IEEE International Conference on Data Mining (ICDM-2005)*, pp. 314–321.

Mitchell, T. (1997). Machine Learning. McGraw-Hill, New York, NY.

- Monge, A. E., & Elkan, C. (1996). The field matching problem: Algorithms and applications. In Proceedings of the Second International Conference on Knowledge Discovery and Data Mining (KDD-96), pp. 267–270 Portland, OR.
- Neal, R. M., & Hinton, G. E. (1998). A view of the EM algorithm that justifies incremental, sparse, and other variants. In Jordan, M. I. (Ed.), *Learning in Graphical Models*, pp. 355–368. MIT Press.
- Needleman, S. B., & Wunsch, C. D. (1970). A general method applicable to the search for similarities in the amino acid sequences of two proteins. *Journal of Molecular Biology*, 48, 443–453.
- Newcombe, H. B. (1988). *Handbook of record linkage: methods for health and statistical studies, administration, and business*. Oxford University Press.
- Newcombe, H. B., Kennedy, J. M., Axford, S. J., & James, A. P. (1959). Automatic linkage of vital records. *Science*, 130, 954–959.

- Papoulis, A., & Pillai, S. U. (2001). Probability, Random Variables and Stochastic Processes (Fourth edition). McGraw-Hill Inc., New York.
- Pasula, H., Marthi, B., Milch, B., Russell, S., & Shpitser, I. (2003). Identity uncertainty and citation matching. In Advances in Neural Information Processing Systems 15. MIT Press.
- Peleg, D. (2000). Approximation algorithms for the Label-CoverMAX and Red-Blue Set Cover problems. In Proceedings of the 7th Scandinavian Workshop on Algorithm Theory (SWAT-2000), LNCS 1851.
- Pereira, F. C. N., Tishby, N., & Lee, L. (1993). Distributional clustering of English words. In Proceedings of the 31st Annual Meeting of the Association for Computational Linguistics (ACL-93), pp. 183–190 Columbus, Ohio.
- Platt, J. (1999a). Fast training of support vector machines using sequential minimal optimization. In Scholkopf, B., Burges, C. J. C., & Smola, A. J. (Eds.), Advances in Kernel Methods - Support Vector Learning, pp. 185–208. MIT Press, Cambridge, MA.
- Platt, J. C. (1999b). Probabilistic outputs for support vector machines and comparisons to regularized likelihood methods. In Smola, A. J., Bartlett, P., Schölkopf, B., & Schuurmans, D. (Eds.), *Advances in Large Margin Classifiers*, pp. 185–208. MIT Press.
- Quinlan, J. R. (1993). *C4.5: Programs for Machine Learning*. Morgan Kaufmann, San Mateo, CA.
- Rabiner, L. R. (1989). A tutorial on hidden Markov models and selected applications in speech recognition. *Proceedings of the IEEE*, 77(2), 257–286.
- Ravikumar, P., & Cohen, W. W. (2004). A hierarchical graphical model for record linkage.

In Proceedings of the 20th Conference on Uncertainty in Artificial Intelligence (UAI-2004).

- Ristad, E. S., & Yianilos, P. N. (1998). Learning string edit distance. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 20(5), 522–532.
- Roy, N., & McCallum, A. (2001). Toward optimal active learning through sampling estimation of error reduction. In *Proceedings of 18th International Conference on Machine Learning (ICML-2001)*, pp. 441–448. Morgan Kaufmann, San Francisco, CA.
- Salton, G., & McGill, M. J. (1983). *Introduction to Modern Information Retrieval*. McGraw Hill, New York.
- Sarawagi, S., & Bhamidipaty, A. (2002). Interactive deduplication using active learning. In Proceedings of the Eighth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD-2002) Edmonton, Alberta.
- Segal, E., Battle, A., & Koller, D. (2003). Decomposing gene expression into cellular processes. In Proceedings of the 8th Pacific Symposium on Biocomputing.
- Seung, H. S., Opper, M., & Sompolinsky, H. (1992). Query by committee. In Proceedings of the ACM Workshop on Computational Learning Theory Pittsburgh, PA.
- Shawe-Taylor, J., & Cristianini, N. (2000). Kernel Methods for Pattern Analysis. Cambridge University Press.
- Shi, J., & Malik, J. (2000). Normalized cuts and image segmentation. *IEEE Transactions* on Pattern Analysis and Machine Intelligence, 22(8), 888–905.
- Singla, P., & Domingos, P. (2005). Object identification with attribute-mediated dependences. In Proceedings of the 9th European Conference on Principles and Practice of Knowledge Discovery in Databases (PKDD-2005) Porto, Portugal.

- Strehl, A. (2002). Relationship-based clustering and cluster ensembles for highdimensional data mining. Ph.D. thesis, The University of Texas at Austin.
- Strehl, A., Ghosh, J., & Mooney, R. (2000). Impact of similarity measures on web-page clustering. In Workshop on Artificial Intelligence for Web Search (AAAI 2000), pp. 58–64.
- Tejada, S., Knoblock, C. A., & Minton, S. (2001). Learning object identification rules for information integration. *Information Systems Journal*, 26(8), 635–656.
- Tejada, S., Knoblock, C. A., & Minton, S. (2002). Learning domain-independent string transformation weights for high accuracy object identification. In *Proceedings of the Eighth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD-2002)* Edmonton, Alberta.
- Tong, S. (2001). Active Learning: Theory and Applications. Ph.D. thesis, Stanford University, Stanford, CA.
- Wagner, R. A., & Fisher, M. J. (1974). The string to string correction problem. Journal of the Association for Computing Machinery, 21, 168–173.
- Wagstaff, K., & Cardie, C. (2000). Clustering with instance-level constraints. In Proceedings of the Seventeenth International Conference on Machine Learning (ICML-2000), pp. 1103–1110 Stanford, CA.
- Wahba, G. (1999). Support vector machines, reproducing kernel Hilbert spaces and the randomized GACV. In Burges, C. J., Schölkopf, B., & Smola, A. J. (Eds.), Advances in Kernel Methods – Support Vector Learning, pp. 69–88. MIT Press.
- Wainwright, M. J., & Jordan, M. I. (2003). Graphical models, exponential families, and variational inference. Tech. rep. 649, Department of Statistics, University of California, Berkeley.

- Wellner, B., McCallum, A., Peng, F., & Hay, M. (2004). An integrated, conditional model of information extraction and coreference with application to citation matching. In *Proceedings of 20th Conference on Uncertainty in Artificial Intelligence (UAI-2004)* Banff, Canada.
- Wettschereck, D., Aha, D. W., & Mohri, T. (1997). A review and empirical evaluation of feature weighting methods for a class of lazy learning algorithms. *AI Review*, 11, 273–314.
- Winkler, W. E. (1993). Improved decision rules in the fellegi-sunter model of record linkage. Tech. rep., Statistical Research Division, U.S. Census Bureau, Washington, DC.
- Winkler, W. E. (1999). The state of record linkage and current research problems. Tech. rep., Statistical Research Division, U.S. Census Bureau, Washington, DC.
- Winkler, W. E. (2002). Methods for record linkage and Bayesian networks. Tech. rep., Statistical Research Division, U.S. Census Bureau, Washington, DC.
- Winkler, W. E. (2005). Approximate string comparator search strategies for very large administrative lists. Tech. rep., Statistical Research Division, U.S. Census Bureau, Washington, DC.
- Winkler, W. E. (2006). Overview of record linkage and current research directions. Tech. rep., Statistical Research Division, U.S. Census Bureau, Washington, DC.
- Witten, I. H., & Frank, E. (1999). Data Mining: Practical Machine Learning Tools and Techniques with Java Implementations. Morgan Kaufmann, San Francisco.
- Xing, E. P., Ng, A. Y., Jordan, M. I., & Russell, S. (2003). Distance metric learning, with application to clustering with side-information. In Advances in Neural Information Processing Systems 15, pp. 505–512 Cambridge, MA. MIT Press.

- Yancey, W. E. (2004). An adaptive string comparator for record linkage. Tech. rep., Statistical Research Division, U.S. Census Bureau, Washington, DC.
- Zhang, Y., Brady, M., & Smith, S. (2001). Hidden Markov random field model and segmentation of brain MR images. *IEEE Transactions on Medical Imaging*, 20(1), 45–57.
- Zhu, J. J., & Ungar, L. H. (2000). String edit analysis for merging databases. In *Proceedings* of the KDD-2000 Workshop on Text Mining.

## Vita

Misha Bilenko was born in Saratov, Russia in 1978. After graduating from Physicotechnical Lyceum in 1995, he decided to go west and found himself in Spokane, WA, where he studied Computer Science and Physics at Whitworth College, graduating *summa cum laude* in 1999. After spending a year back in Russia, he began graduate studies in the Department of Computer Sciences at the University of Texas at Austin in 2000. Still unable to resist the urge to go west, he will next join Microsoft Research in sunny Redmond, WA.

Permanent Address: Department of Computer Sciences Taylor Hall 2.124 University of Texas at Austin Austin, TX 78712-1188 USA

This dissertation was typeset with  $\text{LAT}_{EX} 2\epsilon^{1}$  by the author.

 $<sup>{}^{1}\</sup>text{LAT}_{EX} 2_{\epsilon}$  is an extension of  $\text{LAT}_{EX}$ .  $\text{LAT}_{EX}$  is a collection of macros for  $T_{EX}$ .  $T_{EX}$  is a trademark of the American Mathematical Society. The macros used in formatting this dissertation were written by Dinesh Das, Department of Computer Sciences, The University of Texas at Austin, and extended by Bert Kay, James A. Bednar, and Ayman El-Khashab.