CS 377P Spring 2020
Assignment 3
Due: 10 PM, February 26th, 2020

February 20, 2020

Submissions can be at most 1 day late with 10% penalty.

Some of the problems in this assignment ask you to write code in MATLAB, which is available on the public machines in the CS department. You can also use Octave, which is free software and is similar to MATLAB, or Mathematica.

1. (Iterative solution of linear systems, 15 points) Consider the linear system

   $3x + y = 4$
   $x + 2y = 3$

   (a) (5 points) Write down the recurrence relation that corresponds to solving this system using the Jacobi method, starting with the initial approximation ($x_1 = 0, y_1 = 0$). Use the first equation to refine the approximation for x and the second equation to refine the approximation for y. Express this recurrence as a computation involving matrices and vectors.

   (b) (5 points) Compute the first 10 approximations (x_i, y_i) and plot a 3D plot (x, y, i) in which the z-axis is the iteration number i. Give an intuitive explanation of this 3D plot. You do not need to turn in any code but turn in your plot and explanation.

   (c) (5 points) Repeat these two parts for the Gauss-Seidel method. You can find a description of the Gauss-Seidel method online. It was also described in class.

2. (ODE's, 20 points) Consider the second-order differential equation

   $\frac{d^2y}{dx^2} = -y$

   with initial conditions $y(0) = 0, y'(0) = 1$. The exact solution of this equation is $y = sin(x)$.

   (a) What is the difference equation if we use the backward-Euler method to discretize derivatives? Assume the step size is h.

(b) Discretize the initial conditions to find expressions for the first two terms in the solution to the difference equation. You can approximate $y'(h)$ by $\frac{y_b(h)-y_b(0)}{h}$.

(c) Calculate the solutions to the difference equation in the interval $x = [0, 2\pi]$ for $h = 0.01, 0.1, 0.5, 1.0, 2.0$. Graph each solution together with the exact solution, using a separate graph for each value of h. What trends do you see in your plots? No need to turn in code for the calculations.

(d) Repeat this exercise for the forward-Euler method.

Hint: I used Mathematica on the web (for free) to graph the solutions quickly and make sure that my difference equations were correct. Feel free to use any other way to generate the graphs.

3. (PDE's, 30 points) In this problem, we will solve the one-dimensional diffusion equation, which models how heat spreads through a material of uniform conductivity and similar problems. The diffusion equation is usually written as follows

$\frac{\delta f}{\delta t} = D\frac{\delta^2 f}{\delta x^2}$

The solution $f(x,t)$ depends on both x and t. Assume that we have a rod of length 10 meters, and that the two ends of the rod are kept at a fixed temperature of 0^oC, so $f(0,t) = 0.0$ and $f(10,t) = 0.0$. Assume that initially the temperature in the interior of the rod is $f(x,0) = e^{-4(x-5)^2}$.

The approximate solution $\hat{f}$ can be thought of a two-dimensional array in which $\hat{f}(i,j) \approx f(i\Delta x, j\Delta t)$. Intuitively, $\hat{f}(i,j)$ is the computed solution after j time steps at a spatial position i spatial steps away from the origin.

Compute the array $\hat{f}$ using the following discretization scheme, which discretizes time using Forward-Euler and discretizes space using centered-differences.

$$\begin{aligned} \Delta x &= 0.25 \\ \Delta t &= 0.025 \\ D &= 1 \\ 0 \le x &\le 10 \\ 0 \le t &\le 10 \end{aligned}$$

$\frac{\delta f}{\delta t}\big|_{(i*\Delta x, j*\Delta t)} \approx \frac{\hat{f}(i,j+1)-\hat{f}(i,j)}{\Delta t}$ (Forward-Euler)

$\frac{\delta^2 f}{\delta x^2}\big|_{(i*\Delta x, j*\Delta t)} \approx \frac{\hat{f}(i+1,j)-2*\hat{f}(i,j)+\hat{f}(i-1,j)}{(\Delta x)^2}$ (Centered differences)

(a) Draw the stencil for this discretization scheme. You will find it useful to think of this problem in terms of filling in a matrix whose rows correspond to time-steps and columns correspond to spatial regions in the grid. Notice that the values in row 0 of this matrix are given to you by the initial condition and the values in the first and last columns for all time steps are given by the boundary condition, so the problem is to fill in the interior elements of this matrix.

(b) Use the recurrence equation to find an approximate solution to this problem. Plot the temperature distribution along the rod for different time steps on the same graph (so the x-axis is the distance from the origin in the rod and the y-axis is the temperature). Does this graph jive with your intuition? No need to turn in code.

(c) Increase the time step to 0.050 and repeat the previous steps. Explain your observations briefly.

4. (15 points) Explain the following terms in a few sentences each.

   (a) (4 points) Spatial and temporal locality in memory references

   (b) (6 points) The 3 C's: cold misses, capacity misses, conflict misses

   (c) (4 points) Direct-mapped cache, set-associative cache

   (d) (1 points) Explain briefly why the set size in a set-associate cache does not have to be a power of 2.

5. (20 points) Consider a processor with a two-level memory hierarchy consisting of a single level of cache and main memory, with the following specifications:

   (a) Addresses are 32 bits, and each address corresponds to a word in memory.

   (b) The cache is direct-mapped. The line size is 4 words, and the total cache capacity is 16 words.

   Answer the following questions.

   (a) (5 points) Write down a formula to compute the line number from a given address. Hint: since the line size is 4 words, the first 4 words in memory are in line number 0, the next four are in line 1 etc.

   (b) (5 points) Use this to write down another formula for computing the index into the cache for a given address (the index is sometimes called the block offset).

   (c) (10 points) Consider the following stream of addresses in increasing time order from left to right: 0,2,4,16,3,33,37,41,45,35

   For each address, say whether it will be a hit or a miss in the cache. If it is a miss, classify it as a cold miss, capacity miss, or conflict miss. You must explain your answer for each address in a few sentences (no credit for guessing).