Quiz #1, CS 329E, Spring 2008

*Instructions: Put your name on each page. Show all your work, as I will give partial credit. Do* not *do the extra credit until the other questions are answered.*

Problems:

1. Phrase the following real world problem as a graph-theoretic problem: *You are given a collection of people, and you know which people dislike each other. Your task is to get rid of as few people as possible so that none of the people who are left dislike each other.*

2. Consider an oracle which can answer any question of the following form: *For a given set* $S = \{s_1, s_2, \ldots, s_n\}$ *with* $weight(s_i) = w_i$*,and given integer* $B$*, does there exist a subset of* $S$ *whose total weight sums up to* $B$*?*

   Show how to use this oracle to find the subset with the total weight $B$ from a given input set.

3. Prove by induction that the function $f(n)$ defined by $f(0) = 1$ and $f(n) = 3f(n-1) + n$ if $n > 0$ satisfies $f(n) \geq 3^n$.

4. Indicate, for each of the following statements, if it is True or False.

   (a) $3n^2$ is $O(5n^3 - 5n^2)$

   (b) $ln(n)$ is $O(n/ln(n))$

   (c) $n/ln(n)$ is $O(n)$

   (d) $3n$ is $O(n)$

   (e) $\sqrt{n}$ is $O(\log n)$

   (f) $\sqrt{n}$ is $O(n)$,

   (g) $5n^3$ is $O(n^3 + 200)$.

5. Give a formula for the number of objects described:

   - The number of ways of putting 100 objects in a row.
   - The number of ways of picking 50 objects out of 100.
   - The number of subsets of a set of 17 objects.

**Extra Credit**

1. For the next questions, assume that the menu has 5 possible appetizers, 10 main courses, and 3 deserts.

   - How many different possible meals are there if you must pick one appetizer, one main course, and one desert?
   - How many different meals are there which have a main course and one appetizer *OR* a main course and one desert?
   - How many different meals are there which have two appetizers, one main course, and one desert?

2. Consider the following variant of the rocks game, in which there are three piles of rocks, the objective is to be the person to remove the last rock, and the legal moves are:

   - take one rock from exactly one pile, or
   - take one rock from each of two piles.

   Give a dynamic programming algorithm to determine the winner for this version of the rock game with 50 rocks in each of three piles. Let $M(i, j, k)$ denote the winner when the number of rocks in each of the three piles is $i, j$, and $k$. (Assume that the winner for $0, 0, 0$ is player number 2.)