Factorizing the factorial

Let n be a natural number; let p be a prime;
m = the exponent of p in the prime factorization of $n!$;
s = the sum of the (p-ary) digits of n's representation in base p.

Theorem $\quad m = \dfrac{n-s}{p-1}$

Because of the usual recursive definition of $n!$, an inductive proof over n seems indicated. Since for $n=0$, $m=0 \wedge s=0$, the theorem holds for $n=0$.

Consider now the transition from n to $n+1$; let k be the exponent of p in the prime factorization of $n+1$.

Because $(n+1)! = (n+1)\cdot n!$ and p is prime

(0) $\quad \Delta m = k$.

Because $(n+1) - n = 1$

(1) $\quad \Delta n = 1$.

Because the p-ary representation of $(n+1)$ ends on exactly k zeros

(2) $\quad \Delta s = 1 - k\cdot(p-1)$

Combining (1) and (2) yields

(3) $\quad \Delta \dfrac{n-s}{p-1} = k$.

Combining (0) and (3) yields the induction step, and thus the proof is completed.

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