Should We Teach Formal Methods at All?

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Visions Lecture
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Dedicated

to the memory of Amir Pnueli
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Lecture Subject

- Should we teach formal methods to undergraduates?
- If so, why and how?
What this lecture is not about

• My latest research.

• I find formal methods delightful.

• Tales from a traveling salesman.
Who I am not

- Dana Scott.
- An apologist for formal methods.
- Candidate for any office.
Euclid’s argument for the GCD algorithm

http://aleph0.clarku.edu/~djoyce/java/elements/book
Let $w$ be the predicate: to be a predicate that can not be predicated of itself.

Can $w$ be be predicated of itself?

From each answer its opposite follows. Therefore we must conclude that $w$ is not a predicate.

... The above contradiction, when expressed in Peano’s ideography, reads as follows:

$$w = cls \cap x \varepsilon(x \sim \varepsilon x). \supset: w \in w. =. w \sim \varepsilon w$$
Russel Paradox in Modern Notation

\[ S = \{ x \mid x \notin x \}, \text{ or} \]

for all \( x, x \in S \equiv x \notin x \)

**Proof:** Instantiate \( x \) by \( S \)

\[ S \in S \equiv S \notin S \]
Recreational Math

- Lack of formal notation and symbol manipulation.
- Plenty of pictures, analogies and examples.
- Only common-sense reasoning.
- Proof of specialized cases, omission of boundary cases.

In our teaching of professionals, we do no better.
Cantor Diagonalization

**Theorem:**
There is no 1-1 correspondence between a set and its powerset.
Proof of Cantor’s Theorem

Let $f$ be a 1-1 correspondence from $S$ to its powerset. Let $T$ be the subset of $S$ defined by

$$T = \{x \mid x \notin f(x)\},$$
or

for all $x$, $x \in T \equiv x \notin f(x)$

Instantiate $x$ by $f^{-1}(T)$

$$f^{-1}(T) \in T \equiv f^{-1}(T) \notin f(f^{-1}(T)), \text{ or}$$
$$f^{-1}(T) \in T \equiv f^{-1}(T) \notin T$$
But Computer Science is different!

- Computer science is about programming, not math.

- Our students have poor math background; they will never get it.

- No one else teaches this stuff; text books don’t teach it.

- I can teach this material equally convincingly without using any formalism at all.

- Formal methods just waste a lot of time; you could cover much more material in the same time.

- The only formal methods I like are the ones I don’t see.
Formal methods come in 3 Flavors

- Good
- Bad
- Ugly
Good Formal methods

- Good (effective) formalisms are few in number.
- Goal is not merely correctness, but minimizing mental effort.
- Goal is not merely verification; it is for specification, design, analysis and synthesis.
Our curriculum has become math-phobic\(^1\)

Among 5 most popular books on data structures, each in the neighborhood of 800 pages

Only one book devoted 8 pages to program correctness.

Message to students: muddle through with words, pictures and plausible explanations than make an honest effort at a proof.

Do spend a lot of time muddling through.

How do we explain without invariants

- Heapsort
- Quicksort
- Red-Black Trees
- Scan-Mark Algorithm for Garbage Collection
- ...

Euclid’s GCD algorithm

\[
\begin{align*}
&u, v := x, y \\
&\{u \geq 0, \ v \geq 0, \ u \neq 0 \lor v \neq 0, \ \gcd(x, y) = \gcd(u, v)\} \\

&\text{while } v \neq 0 \text{ do} \\
&\quad u, v := v, u \mod v \\
&\text{od} \\
\end{align*}
\]

\[
\begin{align*}
&\{\gcd(x, y) = \gcd(u, v), \ v = 0\} \\
&\{\gcd(x, y) = u\}
\end{align*}
\]
Extended Euclid Algorithm

Given nonnegative integers $x$ and $y$, where both integers are not zero, there exist integers $a$ and $b$ such that

$$a \times x + b \times y = \gcd(x, y)$$
Extended Euclid Algorithm

invariant: \((a \times x + b \times y = u) \land (c \times x + d \times y = v)\)

\(u, v := x, y; \ a, b := 1, 0; \ c, d := 0, 1;\)

while \(v \neq 0\) do

\[
\{ (a \times x + b \times y = u) \land (c \times x + d \times y = v) \}
\]

\(u, v := v, u \mod v;\)

\(a, b, c, d := a', b', c', d'\) —— what are \(a', b', c', d'\)?

\[
\{ (a \times x + b \times y = u) \land (c \times x + d \times y = v) \}
\]

od
What we must teach well

- Basic algebra: Sets, Functions, Relations, Partial Order
- Basic Logic: Propositional and Predicate Calculus
- Recursion and Induction
- Invariant and well-founded order

For advanced students
- Structural Induction
- Temporal Logic (Thanks to Amir Pnueli)
What we must teach well

- teach the basics relentlessly.
- teach when formal methods are inadequate.
Can UT take the lead?

In formal methods, UT boasts:

- Allen Emerson,
- Robert Van de Geijn,
- Warren Hunt,
- Vladimir Lifschitz,
- J Moore, and
- many others ...
Our courses could emphasize

- writing careful specifications (contracts),
- analyzing specifications, manually and automatically,
- annotating programs with invariants,
- having termination arguments, and
- understanding where formal methods are not applicable.