Comments on Arbeitsblatt 3 from o.Prof.Dr.F.L.Bauer.

by Edsger W. Dijkstra and C.S. Scholten

For the integer function \( m(x, y) \) of the integer arguments \( x \) and \( y \), given by
\[
m(x, y) = \begin{cases} x \geq y & \Rightarrow x = y \rightarrow x+1 \\ x < y & \Rightarrow y - y-1 \end{cases}
\] (1)

it is required to prove that
\[
(x = y \rightarrow y+1 \ [\ x \neq y \rightarrow m(x, m(x-1, y+1))]) = m(x, y)
\] (2).

**Proof.** From (1) follows that the first alternative in the left-hand side of (2) may be replaced by \( x = y \rightarrow m(x, y) \); the proof is complete when we show that the second alternative can be replaced by \( x \neq y \rightarrow m(x, y) \). This latter replacement is allowed as we shall prove
\[
m(x, m(x-1, y+1))) = m(x, y)
\] (3)

From (1) follows
\[
x \geq y \Rightarrow (m(x, x) = m(x, y))
\] (4)

From (1) it also follows that the only possible values of \( m(x-1, y+1) \) are \( x \) and \( y \). If \( m(x-1, y+1) = y \), (3) holds trivially. Otherwise we have \( m(x-1, y+1) \neq y \), which implies --on account of (1)--
\[
m(x-1, y+1) = x \text{ and } x-1 \geq y+1
\] (5)

As (5) implies \( x \geq y \), we conclude by (4) and (5) that (3) then holds as well. (End of proof.)

The above proof has been given, because 33(1) lines of formal text, as dedicated to a proof of this theorem in said Arbeitsblatt 3, is to our tastes a bit unwieldy. We appreciate that the proof in said Arbeitsblatt 3 has been conducted in terms of a limited repertoire of operations that seem suitable for mechanization. When, however, the length of such proofs and their obvious tedium are presented --as is often the case-- as conclusive evidence for the necessity of such mechanizations, it should be clear that at least the above example has failed to convince us of such necessity.

Plataanstraat 5
NL-4565 NUIJEN
The Netherlands

prof.dr.Edsger W.Dijkstra
Burroughs Research Fellow