Copyright Notice

The following manuscript

EWD 608: An elephant inspired by the Dutch National Flag

is held in copyright by Springer-Verlag New York.

The manuscript was published as pages 264–267 of

Edsger W. Dijkstra, Selected Writings on Computing: A Personal Perspective,

Reproduced with permission from Springer-Verlag New York. Any further reproduction is strictly prohibited.
An elephant inspired by the Dutch National Flag.

Encouraged by the success of EWD607, we now embark upon the analysis of a more intricate elephant. We start with a cyclic arrangement of 3 + 3 mosquitoes. Three main mosquitoes, called R(ed), W(hite), and B(lue) respectively, and three buffer mosquitoes RW, WB, and BR, in between:

\[ R \rightarrow RW \rightarrow W \rightarrow WB \rightarrow B \rightarrow BR \rightarrow R \]

The buffer mosquitoes are quite simple, e.g.:

\begin{verbatim}
RW: begin channel W;
    begin channel R; buf: pebble;
    do R?(buf) \rightarrow W!(buf) od

end
\end{verbatim}

When its (input) channel with R ceases to exist, R?(buf) will become false, and block exit will cause termination of the existence of the (output) channel with W.

Each of the main mosquitoes has three "bags of pebble", named "r(ed)", "w(hite)", and "b(lue)". The R mosquito must collect in its bag called "r" all red pebbles in the system; its "foreign" pebbles it transmits, one at a time, via the buffer mosquito RW, first emptying its blue bag because its blue pebbles -- that have to reach their destination via W -- have to travel the longer distance. The arrangement is worth investigating because we expect problems with the proof of termination.

The solution that I am proposing has also a starting problem, but I am not going to divulge that now: I hope that that difficulty emerges "naturally" from a systematic analysis of our system.

mosquito R:

\begin{verbatim}
begin channel BR;

x, y: pebble; r, w, b: bag of pebble;
\end{verbatim}
proc accept: if non BR?(y) → skip [ BR?(y) → place fi corp;  
proc place: if white(y) → w := w ≠ y [ red(y) → r := r ≠ y fi corp;  
 r, w, b := "initial values" {R3}; 
begin channel RW;  
do card(b) > 0 → x := any(b); b := b ≤ x;  
    RW!(x); accept 
od {R2};  
do card(w) > 0 → x := any(w); w := w ≤ x;  
    RW!(x); accept 
od  
end {R1};  
do BR?(y) → place od {R0}  
end  

(and cyclicly).  

Note: "card" — short for "cardinality"— denotes "number of elements in".  
(End of note.)

We assume that --by some magic, not to be discussed here-- BR (the text of which starts with "begin channel BR") and R (the text of which starts with "begin channel R") perform the entry to their outer blocks simultaneously, thereby establishing the channel between them (which will be used only as an input channel to R). When the three input channels to the main mosquitoes have been established, the six inner blocks will be entered --pairwise simultaneously, but now R paired with RW -- and the output channels for the main mosquitoes have been established. (This is very informal and intuitive, but OK for the moment: if coded wrongly, such paired block entries can, of course, create a glorious deadlock.)

Let us now study mosquito R backwards. My final goal is to establish proper termination with

R0: card(b) = card(w) = 0 and y-tail(R0) is empty,

i.e. mosquito R has to terminate with red pebbles only when nothing will be sent to it anymore; with "y-tail(Ri)" I denote the sequence of y-values still to be absorbed in stage Ri before BR?(y) turns definitely false.

The first step is to investigate the transition from R1 to R0. Termination of the repetitive construct in between guarantees non BR?(y), i.e. guarantees that y-tail(R0) is empty; infinite repetition is excluded by

y-tail(R1) is finite .
Because \( \text{card}(b) = \text{card}(w) = 0 \) does not follow from "non BB" it had better hold at \( R1 \) and be kept invariant by "place". Keeping \( \text{card}(w) = 0 \) invariant by "place" implies the absence of white pebbles in the tail, avoiding abortion implies the absence of blue ones, and we find for \( R1 \):

\[
R1: \quad \text{card}(b) = \text{card}(w) = 0 \quad \text{and} \quad y\text{-tail}(R1) \text{ is finite and red only}
\]

Note The condition "finite and red only" is satisfied by the empty tail. (End of note.)

The next step is to investigate the transition from \( R2 \) to \( R1 \). Because \( \text{card}(b) = 0 \) does not follow from "non BB", we require it at \( R2 \); exclusion of abortion taken into account:

\[
\text{card}(b) = 0 \quad \text{and} \quad y\text{-tail}(R2) \text{ contains no blue pebbles}.
\]

We have to impose more, because we have also to guarantee

\[
\text{card}(w) = 0 \quad \text{and} \quad y\text{-tail}(R1) \text{ is finite and red only}.
\]

Termination guarantees \( \text{card}(w) = 0 \) and is guaranteed by

\[
y\text{-tail}(R2) \text{ is finite}.
\]

(For the variant function we can take: \( \text{card}(w) + \text{number of white pebbles in } y\text{-tail} \).)

But how do we guarantee that \( y\text{-tail}(R1) \) is red only?

Let us define for a finite tail without blue pebbles

- if tail contains no white pebbles: \( \text{slack} = -1 \)
- if tail contains white pebbles: \( \text{slack} = \text{the total number of red pebbles preceding the last white one} \)

and let us consider the relation \( \text{card}(w) > \text{slack} \); then

1) \( \text{card}(w) = 0 \) implies that the finite tail is all red

2) \( \text{card}(w) > \text{slack} \) is an invariant for the repeatable statement from \( R2 \) to \( R1 \); because \( \text{card}(w) \geq 0 \) by definition, this is obvious if the resulting tail has no white pebbles, otherwise

2a) \( y \) has been white, in which case both \( \text{card}(w) \) and \( \text{slack} \) remained unchanged

2b) \( y \) has been red, in which case both \( \text{card}(w) \) and \( \text{slack} \) have been decreased by 1.

Hence, collecting all our requirements, we deduce
R2: \( \text{card}(b) = 0 \) and \( \text{y-tail}(R2) \) is finite, without blue pebbles and \( \text{card}(w) > \text{slack}(R2) \).

For the transition from \( R3 \) to \( R2 \), infinite repetition is excluded a priori, abortion is excluded by the absence of blue pebbles in the tail, the invariant relation that does the trick is
\[
\text{card}(b) + \text{card}(w) > \text{slack}
\]
and we find for \( R3 \)

R3: \( \text{y-tail}(R3) \) is finite, without blue pebbles and \( \text{card}(b) + \text{card}(w) > \text{slack}(R3) \).

Taking the finiteness for a moment for granted, we see that
1) the absence of blue pebbles in the y-tail is guaranteed (because \( R \) does not transmit red pebbles, and cyclicly)
2) \( \text{slack}(R3) \geq 0 \) (because \( R \) does transmit blue pebbles, if any, before white ones, if any, and cyclicly.)

Hence, a safe starting state is: each mosquito with at least one, foreign pebble! The complication at the start has, indeed, shown up nicely.

Termination was more easily demonstrated than originally feared.
1) Mosquito \( R \) will generate in its \( x \)-sequence an a priori bounded number of blue pebbles.
2) In the same way, mosquito \( B \) will only generate in its \( x \)-sequence an a priori bounded number of white pebbles.
3) Equating the \( x \)-output of \( B \) with the \( y \)-input of \( R \), we conclude that mosquito \( R \) will only receive a bounded number of white pebbles.
Combining 1) and 3) we conclude that mosquito \( R \) will only generate a finite \( x \)-sequence.

The proof of total conservation of pebbles is left to the reader.

---

Platoanstraat 5  
NL-4565 NUENEN

The Netherlands

prof.dr.Edsger W.Dijkstra  
Burroughs Research Fellow