My mother's contribution to Honsberger's collection.

In Fig. 1 we have an equilateral triangle ABC with a circumscribed rectangle PQRC. Prove that the areas $x$, $y$, and $z$ of the three rectangular triangles in Fig. 1 satisfy $x + y = z$.

Subject triangle APC to an anti-clockwise rotation of 60° around A; in Fig. 2, $P'$ and $C'$ are the images of $P$ and $C$. Subject triangle BRC to a clockwise rotation of 60° around B; in Fig. 2, $R''$ and $C''$ are the images of $R$ and $C$. Because all three triangles are rectangular, the circle with diameter AB is the circumscribed circle of all three. Because our rotations were over 60°, $\angle P'AQ = \angle R''BQ = 120°$. Hence, $\text{arc } P'AQ = \text{arc } R''BQ = \frac{2}{3}$ of the circle's circumference; hence, so is $\text{arc } P'R''$, in other words: in Fig. 2 triangle $P'QR''$ is equilateral.
Because in an equilateral triangle the centroid coincides with the circumcentre, the centroid of $P'QR'$, called $M$, lies on the diameter $AB$. Therefore, with equal weights in its vertices, triangle $P'QR'$ is in balance when supported by a horizontal axis $AB$. From the fact that the torque caused by the weight at $Q$ is compensated by the sum of the torques caused by the weights at $P'$ and $R''$ respectively, $z = x + y$ immediately follows.

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The problem stated in the first paragraph occurred in a series of geometrical problems I received from Ross A. Homsberger. After having sent a copy to my mother, mrs. B.C. Dijkstra-Kluiver, I received (among others) by returning mail the above argument, which I think too beautiful not to be recorded.

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