A (new?) proof of a theorem of Euler's on partitions.

Euler's theorem. For any natural number $N$, the number of bags of (not necessarily distinct) odd natural numbers whose sum is $N$ equals the number of sets of (distinct) positive integers whose sum is $N$.

Proof. In this proof:
1. $c$'s stand for positive integers,
2. $q$'s stand for odd natural numbers, and
3. $t$'s stand for powers of 2.

Euler's theorem follows from a 1-1 correspondence between bags of $q$'s and sets of $c$'s with the same sum.

In order to construct the bag of $q$'s corresponding to a given set of $c$'s, we observe that for each $c$ the factorization $c = t \cdot q$ is unique. For each $c$ in the set we put, with $c = t \cdot q$, $t$ instances of $q$ into the bag. The result is a bag of $q$'s with the same sum.

In order to construct the set of $c$'s corresponding to a given bag of $q$'s, we observe that each natural number $f$ is uniquely the sum of distinct $t$'s. For a $q$ with $f$ occurrences in the bag, we put into the set the $c$'s of the form $t \cdot q$ for those distinct $t$'s whose sum equals $f$. The result is a set of (distinct) $c$'s.
with the same sum.

Grouping the i's in the set by largest odd divisor we see that the two transformations are each other's inverse. (End of Proof.)

I found the above proof shortly after I had firmly decided to discard Ferrer diagrams and similar pictorial "aids." I asked myself how I could derive simply a bag of q's in a sum-preserving manner from a given set of i's. Since I was heading for a bag (in which multiple occurrences are allowed) I investigated how I could transform in a sum-preserving manner the individual i's into bags of q's. With the above result.

Plataanstraat 5
5671 AL NUENEN
The Netherlands

27 April 1981
prof. dr. Edsger W. Dijkstra
Burroughs Research Fellow

P.S. By distributing the above I learned that the proof is already known.

EWD.