On equality of propositions.

After the observation that equality of propositions seems to be underused in mathematical reasoning, I discovered that I myself was — perhaps not surprisingly — only vaguely familiar with all sorts of equalities. This note — which is not deep at all — is written to remedy that situation. I shall not repeat that ∧ and ∨ are symmetric, associative and mutually distributive; neither shall I repeat de Morgan's Laws.

The following expressions (E0.6) are all equal.

\[(E0.0) \quad \top\]
\[(E0.1) \quad \top \top\]
\[(E0.2) \quad \top \top\]
\[(E0.3) \quad \top \top\]
\[(E0.4) \quad \top \top\]
\[(E0.5) \quad \top \top\]
\[(E0.6) \quad \top \top (\top \top \top)\]
\[(E0.7) \quad \top \top (\top \top \top)\]

With the possible exception of the last two, (the so-called "Laws of Absorption") this is very familiar ground. Also expressions (E1.0) are all equal.

In the following, = has the lowest binding power
(E1.0)  \( P = Q \)
(E1.1)  \( \neg P = \neg Q \)
(E1.2)  \( (P \lor \neg Q) \land (\neg P \lor Q) \)
(E1.3)  \( (P \land Q) \lor (\neg P \land \neg Q) \)

Also expressions (E2.1) are all equal; since (E2.0) is symmetric, they occur in pairs.

(E2.0)  \( P \lor Q \)
(E2.1)  \( P \lor \neg Q = P \)
(E2.2)  \( \neg P \lor Q = Q \)
(E2.3)  \( P \land \neg Q = \neg Q \)
(E2.4)  \( \neg P \land Q = \neg P \)
(E2.5)  \( P \land (R \lor \neg Q) = (P \land R) \lor \neg Q \)
(E2.6)  \( (\neg P \lor R) \land Q = \neg P \lor (R \land Q) \)

The last two were absolutely new for me; their discovery — in a completely different context— provided the incentive to write this note.

Platanistraat 5
5671 AL NUENEN
The Netherlands

5 October 1981
prof. dr. Edsger W. Dijkstra
Burroughs Research Fellow