A short sequel to EWD842

I think I have now found a formulation of the required generalization of Leibniz's Rule. As it stood it described how a new formula may be generated "in the presence of \( P \equiv R \)."

The generalization describes how a new formula may be generated "in the presence of \( A \lor B \)," essentially by interpreting the presence of \( A \lor B \) as the presence of at least 1 from the bag \( \{A, B\} \).

If \( F \) may be generated (presumably in the presence of \( A \) but) without using \( B \) and \( F \) may be generated (presumably in the presence of \( B \) but) without using \( A \), then \( F \) may be generated in the presence of \( A \lor B \).

In particular — but this is only a special case — if \( G \) may be generated using \( A \) but not \( B \), whereas we don't succeed generating \( G \) using \( B \) but not \( A \), we can form a formula \( F \) that satisfies the second requirement as well, by choosing for \( F \) the formula \( B \lor G \).

Note. In EWD842 — and I remember that I hesitated when I did so! — I defined the disjunction \( \lor \) (and consequently the conjunction \( \land \)) only on bags containing at least 2 operands. The proper extension is of course to define the disjunction (and consequently the conjunction) of a bag of 1 operand as that operand, (and to define the disjunction of the empty bag in view of Theorem 5 as "black", and its conjunction as "black"). The equivalence on a bag of 1 operand should be defined as that operand, the equivalence on an empty bag should, in view of Theorem 0 be defined as "black".

Leibniz's Rule in the presence of an equivalence is now recursively defined according to the syntax.
<equivalence> ::= <disjunction> {≡<disjunction>}^* 
<disjunction> ::= <term> {∨ <term>}^* 
<term> ::= {¬}* <primary> 
<primary> ::= <predicate variable> 
          | (<equivalence>)

Remark At least the associativity of \(\equiv\) could
have been expressed syntactically by
<equivalence> ::= <disjunction>
          | <equivalence> \(\equiv\) <equivalence>,

etc. (End of Remark.)
(End of Note.)

Note that we did not interpret the presence
of \(A \lor B\) as a promise that eventually at
least 1 formula from the bag \{A, B\} can be
generated! In a common application, \(A \lor B\)
takes the form \(A \lor \neg A\) — which we can generate
in view of Theorem 2 for any \(A\) — , and the
rejected interpretation would exclude the existence
of undecidable formulae — i.e. formulae \(A\) such that
neither \(A\) nor \(\neg A\) can be generated — .

The above use of the presence of \(A \lor \neg A\) in
the case of undecidable \(A\) does not bother me
at all. In that case we can add either \(A\) or
\(\neg A\) to our set of Axioms, thus creating two
different systems. Our usage of \(A \lor \neg A\) generates
what those two systems have in common.
So I prefer not to be bothered; undoubtedly I
am very naive, but I definitely prefer to remain so
as long as possible.

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