A short sequel to EWD863

In EWD863, I left at the end the derivation of de Morgan's Laws as an exercise to the reader. The following proof, however, is too beautiful to remain unrecorded. I recall - with numbers as in EWD863 - the axioms

\[ P \land Q \equiv P \equiv Q \equiv P \lor Q \]  \hspace{1cm} (9)

\[ P \lor \neg Q \equiv P \lor Q \equiv P \]  \hspace{1cm} (14)

and the theorems

\[ \neg \neg Q \equiv Q \]  \hspace{1cm} (19)

\[ \neg (P \equiv Q) \equiv \neg P \equiv Q \]  \hspace{1cm} (20)

From (14), we derive with $P := \neg P$ and $P, Q := Q, P$ respectively:

\[ \neg P \lor \neg Q \equiv \neg P \lor Q \equiv \neg P \]  \hspace{1cm} (14a)

\[ Q \lor \neg P \equiv Q \lor P \equiv Q \]  \hspace{1cm} (14b)

From those two with Leibniz's Principle (and symmetry of $\lor$ and $\equiv$)

\[ \neg P \lor \neg Q \equiv \neg P \equiv Q \equiv P \lor Q \]  \hspace{1cm} (14c)

from that one with (20)

\[ \neg P \lor \neg Q \equiv \neg (P \equiv Q \equiv P \lor Q) \]  \hspace{1cm} (20a)

and finally with (9)
\[ \neg P \lor \neg Q \equiv \neg (P \land Q) \] \hspace{1cm} (25)

With (19) and \[ \neg P \equiv \neg \neg P \], which is a syntactic descendant of \[ P \equiv P \] -

\[ \neg P \lor \neg Q \equiv \neg (P \lor Q) \] \hspace{1cm} (26)

follows readily from (25).

Austin, 2 Dec. 1984

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PS. The single substitution \( P, Q \leftarrow \neg Q, \neg P \) into (14), yielding

\[ \neg Q \lor \neg P \equiv \neg Q \lor P \equiv \neg Q \]

would have sufficed in subsequent combination with (14).

(End of PS.)