An improvement on EWD912

In my final comments on EWD912 I suggested that Theorem 5.0 would admit a shorter proof. Here is a suggestion.

**Proof Th. 5.0** With \( p \) enjoying some type of conjunctivity, \( p \) is monotonic; hence - Lemma 5.8 - \( h \) is monotonic.

In order to show that \( h \) is conjunctive over some \( V \), i.e.

\[
[h.(\forall x: x \in V : x) \equiv (\forall x: x \in V : h(x))]
\]

we show that either side implies the other.

(i) Because \( h \) is monotonic, we have

\[
[h.(\forall x: x \in V : x) \equiv (\forall x: x \in V : h(x))]
\]

(ii) To show the implication in the other direction, it suffices to show in view of (11) that

\[
[(\forall x: x \in V : h(x) \equiv p((\forall x: x \in V : x), (\forall x: x \in V : h(x)))]
\]

or, since quantification distributes over pair forming

\[(14') \quad [(\forall x: x \in V : h(x) \equiv p((\forall x: x \in V : (x, h(x)))]
\]

To this end we construct a bag \( W \) of predicate pairs by
(16') \((X, Y) \in W \equiv X \in V \land [Y \equiv h.X]\)

and observe that – see Lemma to be inserted in Ch. 3 – \(V\) and \(W\) are of the same junctivity type since \(h\) is monotonic. Hence, suffices to show (14') under the assumption that \(f\) is conjunctive over \(W\).

\[
\begin{align*}
(AX : X \in V : h.X) \\
= \{\text{(10)}\} \\
(AX : X \in V : f.(X, h.X)) \\
= \{\text{one-point rule}\} \\
(AX : X \in V : (AY : [Y \equiv h.X] : f.(X, Y))) \\
= \{\text{(15')}\} \\
(AX, Y : (X, Y) \in W : f.(X, Y)) \\
= \{f \text{ conjunctive over } W\} \\
f.(AX, Y : (X, Y) \in W : (X, Y)) \\
= \{\text{(16')}\} \\
f.(AX : X \in V : (AY : [Y \equiv h.X] : (X, Y))) \\
= \{\text{one-point rule}\} \\
f.(AX : X \in V : (X, h.X))
\end{align*}
\]

(End of Proof Th. 5.0)

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