On a problem posed by M.R. Khalil (for the record)

For some positive integers $N$ and $L$ it is asked to generate -print- in lexical order the integer sequences $f(i:0 \leq i < N)$ satisfying

(0) $(A_i: 0 \leq i < N: 0 \leq f_i < L)$
(1) $f$ is ascending, i.e. $(A_{i,j}: 0 \leq i < j < N: f_i \leq f_j)$

An $f$-sequence containing $n$ different values $(1 \leq n \leq L)$ can be represented by two sequences of length $n$, $v$ (of values) and $fr$ (of frequencies) satisfying

(2) $(A_i: 1 \leq i \leq n: 0 \leq v_i < L)$
(3) $v$ is ascending
(4) $(A_{i,j}: 1 \leq i \leq n: 1 \leq fr_i)$
(5) $(\sum_{i: 1 \leq i \leq n: fr_i}) = N$

and coupled to $f$ by

(6) $(A_{i,j}: 1 \leq i \leq n: (E_{j,k}: (A_{k,j}: j \leq k < j + fr_i: f_k = v_i)))$

Since $n \leq \min(N,L)$, $n$ is at most $N$ and may be much smaller. From given $v$- and $fr$-sequences, the corresponding $f$-sequence is generated by the following (trivial) block

```
"print f": " var i : int; i=0
  do i\neq n -> " var j : int; j=0; i=i+1
    do j\neq fr_i -> print(v_i); j=j+1 od ]
  od
]
```
The program is

\[\text{\| \text{var} \ n: \text{int}; \ \text{var} \ v, tfr: \text{array of int} }\]
\[; \ n:=1; \ v.1:=0; \ tfr.1:=N \]
\[; \ \text{do} \ n \neq 0 \rightarrow \]
\[; \ \text{\| \text{var} \ tv, tfr: \text{int}; \ "print f" }\]
\[; \ \text{if} \ v.n < L-1 \rightarrow \ tfr:=1 \]
\[\rightdownarrow \ v.n = L-1 \rightarrow \ tfr:=fr.n + 1; \ n:=n-1 \]
\[\rightdownarrow \{ \text{n}>0 \rightarrow \ v.n < L-1; (S2:1 \leq i \leq n; fr.i) + tfr = N+1 \} \]
\[; \ \text{if} \ n=0 \rightarrow \ \text{skip } \{ \text{successor construction aborted} \} \]
\[; \ n>0 \rightarrow \ tv:=v.n + 1 \{ tv<\text{L} \} \]
\[; \ \text{if} \ fr.n > 1 \rightarrow fr.n:=fr.n-1; \ n:=n+1 \]
\[\rightdownarrow \ fr.n = 1 \rightarrow \ \text{skip} \]
\[\rightdownarrow \ v.n := tv; \ fr.n := tfr \]
\[\| \]
\[\| \]

The point of the \((v,fr)\)-representation is that the lexical successor can be constructed without repetition. The point of the above program is the separate successive determinations of the "top frequency" and the "top value".

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