Addendum to EWD969

(To be inserted after "(End of Proof 5.6)"
at EWD969-16.)

Remark Theorem 5.6 can be extended. Let \( f \)
be the strongest solution of
\[
X: [p.(X,Y) \equiv X]
\]
i.e. our formal knowledge about \( f \) is captured by
\[
(20') \quad [p.(f.Y,Y) \equiv f.Y]
\]
\[
(21') \quad [p.(X,Y) \Rightarrow X] \Rightarrow [f.Y \Rightarrow X]
\]

Then, analogous to the conclusion \([g.P \equiv Q]\),
Theorem 5.6 allows us to conclude \([f.Q \equiv P]\),
i.e. \((P,Q)\) is a solution of
\[
(X,Y): [(f.Y, g.X) \equiv (X,Y)]
\]
We shall now show that \((P,Q)\) is the strongest
solution of the above equation, i.e. we shall show
\[
[(f.Y, g.X) \equiv (X,Y)] \Rightarrow [(P,Q) \Rightarrow (X,Y)]
\]
To this end we observe for any \(X,Y\) satisfying
the antecedent
\[
[(P,Q) \Rightarrow (X,Y)]
\]
\[
\Leftrightarrow \quad \{ (19) \}
\]
\[
[p.(X,Y) \Rightarrow X] \land [q.(X,Y) \Rightarrow Y]
\]
\[
\Leftrightarrow \quad \{ \text{the antecedent equivales} \quad [f.Y \equiv X] \land [g.X \equiv Y] \}
\]
\[
[p.(f.Y) \Rightarrow f.Y] \land [q.(X, g.X) \Rightarrow g.X]
\]
\[
\Leftrightarrow \quad \{ (20) \text{ and } (20') \}
\]
true
The next theorem will show that above \( f \) and \( g \) are monotonic. The theorem of Knaster-Tarski is therefore applicable and we have therefore that \((P,\mathcal{D})\) is also the strongest solution of

\[
(X,Y) : [(f.Y, g.X) \Rightarrow (X,Y)]
\]

Note how in the above we have proved the weaker implication, i.e. the one with the stronger antecedent: the middle step in the above proof makes essential use of the equivalence. (End of Remark.)

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