Simplifying a proof in our book

In [0], pp. 66-69, we show how the conditional distribution of $\wedge$ over $\forall$ can be derived from the one-point rule and the other axioms. I dragged sets into the picture, for which misbehaviour I apologize; here is a simpler argument. Representing "the non-empty range" for the dummy x by $r.x \vee [x=y]$ we show

$$[\langle \forall x : r.x \vee [x=y] : t.x \wedge Q \rangle \equiv \langle \forall x : r.x \vee [x=y] : t.x \rangle \wedge Q]$$

To this end we observe

$$\begin{array}{ll} & \langle \forall x : r.x \vee [x=y] : t.x \wedge Q \rangle \\ = & \quad \{\text{splitting the term}\} \\ & \langle \forall x : r.x \vee [x=y] : t.x \rangle \wedge \langle \forall x : r.x \vee [x=y] : Q \rangle \\ = & \quad \{\text{see } (*) \text{ below}\} \\ & \langle \forall x : r.x \vee [x=y] : t.x \rangle \wedge Q \quad . \end{array}$$

(*) We observe

$$\begin{array}{ll} & \langle \forall x : r.x \vee [x=y] : Q \rangle \\ = & \quad \{\text{splitting the range}\} \\ & \langle \forall x : r.x : Q \rangle \wedge \langle \forall x : [x=y] : Q \rangle \\ = & \quad \{\text{one-point rule}\} \\ & \langle \forall x : r.x : Q \rangle \wedge Q \\ = & \quad \{\text{see } (**) \text{ below}\} \\ & Q \quad . \end{array}$$

(**) We observe

$$\begin{array}{ll} & [Q \Rightarrow \langle \forall x : r.x : Q \rangle] \\ = & \quad \{\Rightarrow \text{ distributes -like } \vee\text{- over } \forall \text{ in consequent}\} \end{array}$$

$$
\begin{aligned}
& [\langle\forall x : r.x : Q \Rightarrow Q\rangle] \\
= \quad & \{\text{pred. calc}\} \\
& [\langle\forall x : r.x : true\rangle] \\
= \quad & \{\text{pred. calc., e.g. [0], p.66, (90)}\} \\
& [true] \\
= \quad & \{\text{pred. calc.}\} \\
& true \quad .
\end{aligned}
$$

[0] Edsger W. Dijkstra & Carel S. Scholten "Predicate Calculus and Program Semantics", Springer-Verlag New York Inc., 1990

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