Monotonic demonstranda and dummy introduction

When we have to prove

\([f.\, \text{exp}]\)

for some \(\text{exp}\) and some monotonic \(f\), it can help to know the following theorem.

**Theorem**  For monotonic \(f\)

\((0)\) \(\[f.\text{exp}\] \equiv \langle \forall z: \text{[exp} \Rightarrow z]: [f.\, z]\rangle\) and

\((1)\) \(\[f.\text{exp}\] \equiv \langle \exists z: \text{[z} \Rightarrow \text{exp}]: [f.\, z]\rangle\).

A reason to use \((0)\) is that \(\text{[exp} \Rightarrow z]\) is the form of expression in which we can manipulate \(\text{exp}\). An example is given in EWD1118.

A reason to use \((1)\) is that \(\text{[z} \Rightarrow \text{exp}]\) is the form of conclusion we can draw about \(\text{exp}\); if it exists, the strongest \(z\) satisfying \(\text{[f} . z]\) is a good candidate for a witness. An example is given in EWD1116.

This theorem is very simple, very general and probably equally applicable and useful. Why did it take me a lifetime to formulate it?

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