Counting characters

In [0], "factors" are introduced in relation algebra in the usual fashion. I quote the definition of the binary operator \ (pronounced "under"):

(7) \( R \subseteq S \setminus T \equiv S \circ R \subseteq T \)

Instantiating (7) with \( R := S \setminus T \) gives — since \( E \) is reflexive — the "rule of cancelation"

(9) \( S \circ S \setminus T \subseteq T \)

(In [0], other identifiers are used; I have followed its convention of giving \( \setminus \) a higher binding power than \( \circ \).)

Four pages later, the operator \( \setminus \) is used to prove — about the reflexive transitive closure — \( R \circ R^* = R^* \circ R \). The ping-pong argument starts with

\[
R \circ R^* \subseteq R^* \circ R
\]

\[
\equiv \{ \text{factors: (7)} \}
\]

\[
R^* \subseteq R \setminus (R^* \circ R)
\]

\[
\subseteq \ldots \ldots \ldots
\]

The proof contains a second reference to (7) and 2 references to (9).

* * *

Instead of constructing the expression
\( R \setminus (R^* \circ R) \), we could have named it, say \( K \); defining \( K \) would have required

\[ (7') \quad X \in K \equiv R \circ X \in R^* \circ R \setminus \]

and instantiating this with \( X := K \) gives the tailored "rule of cancelation"

\[ (9') \quad R \circ K \in R^* \circ R \setminus \]

The ping-pong argument would then start

\[ R \circ R^* \in R^* \circ R \setminus \]

\[ \equiv \{ (7') \text{ with } X := R^* \} \]

\[ R^* \in K \setminus \]

\[ \equiv \quad \ldots \ldots \]

\[ * \quad * \quad * \]

The introduction of a new identifier \( K \) and the 20 characters needed to formulate \((7')\) and \((9')\) are an investment, but worth the price, because each time we replace \( R \setminus (R^* \circ R) \) by \( K \), we save 7 characters, and the complete proof in [0] presents that opportunity 13 times, and after all \( 13 \times 7 - 20 = 71 \) times. It is significant:

\[ I \cup R \cup R \setminus (R^* \circ R) \circ R \setminus (R^* \circ R) \setminus \]

\[ I \cup R \cup K \circ K \in K \setminus \]

is much less pleasant to deal with than

\[ I \cup R \cup K \circ K \in K \setminus \]
Excluding the hints, the formulae of the proof offering the 13 opportunities consist of 195 characters. (And after all, 195 - 91 = 104)

Why do I spend an evening writing a technical note whose mathematical content is absolutely nil? I think because I wholeheartedly agree with the authors of [0] when they write on pg 1:

"Second, it is not sufficiently recognised that formal methods must combine precision with conciseness."

Amen.


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