A theorem about "factors" perhaps worth recording.

Let \( \setminus \) be defined by

(0) \[ [x; y \Rightarrow z] \equiv [y \Rightarrow x \setminus z] \]

(0) defines \( x \setminus z \) as the weakest solution of

\( y: [x; y \Rightarrow z] \)

and yields with instantiations \( y := x \setminus z \) and
\( z := x; y \) respectively

(1) \[ [x; x \setminus z \Rightarrow z] \]

(2) \[ [y \Rightarrow x \setminus (x; y)] \]

(We have given \( \setminus \) a higher binding power
than \( ; \).)

About the transpose \( \sim \) (prefix) - which
others call the converse \( \circ \) (postfix) - I shall
use the Dedekind Law

(3) \[ [x; y \wedge z \Rightarrow x; (y \wedge \sim x; z)] \]

We shall now prove

(4) \[ [p; q \wedge \sim p \setminus r \Rightarrow p; r] \]

To this end we observe for any \( p, q, r \)

\[ p; q \wedge \sim p \setminus r \]
\[ \Rightarrow \{ (3) with x, y, z := p, q, \sim p \setminus r \} \]
\[ p; (q \land \lnot p; \lnot p) \]
\[ \Rightarrow \{\text{monotonicities}, \ (2) \text{ with } x, z := \lnot p, r \} \]
\[ p; r \]

I used (4) to prove the theorem of section 2.3 of "A Graphical Calculus" by Sharon Curtis and Gavin Lowe, Oxford University Computing Laboratory, Parks Road, Oxford, OX1 3QD; the formulation of (4) was triggered by their note.

An alternative formulation of (4) that incorporates the antimonotonicity of \( \lnot \) in its left argument is

\[ (5) \quad [\lnot p \Rightarrow s] \Rightarrow [p; q \land s \Rightarrow p; r] \]

I think the theorem is worth recording though not worth remembering.

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