A simple geometrical theorem I did not know

Given:

\[ AE = \alpha \cdot AC \]
\[ ED = \beta \cdot EB \]
\[ FD = \gamma \cdot FC \]

Denoting with \((PQR)\) the area of \(\Delta PQR\), we observe

\[(ADB) = \gamma \cdot (ACB) \quad \text{and also} \]
\[(ADB) = (1-\beta) \cdot (AEB) \]
\[= (1-\beta) \cdot \alpha \cdot (ACB) \]

from which we conclude \(\gamma = (1-\beta) \cdot \alpha \).

[The theorem we used thrice—say \((PQR) \cdot \frac{RS}{QR} = (PRS)\)—is no more than adding metric to—see EWD1221b—

\[ R \neq S \wedge \text{col} \cdot R \cdot S \cdot Q \wedge \text{tri} \cdot R \cdot Q \cdot P \Rightarrow \text{tri} \cdot R \cdot S \cdot P \]

The theorem proved in this note is of no importance; it is recorded here because I don't remember this proof technique from my school-days.

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