A simple geometrical theorem I did not know

Given:

\[ AE = \alpha \cdot AC \]
\[ ED = \beta \cdot EB \]
\[ FD = \gamma \cdot FC \]

Denoting with \((PQR)\) the area of \(\Delta PQR\), we observe

\[ (ADB) = \gamma \cdot (ACB) \]

and also

\[ (ADB) = (1-\beta) \cdot (AEB) \]
\[ = (1-\beta) \cdot \alpha \cdot (ACB) \]

from which we conclude \( \gamma = (1-\beta) \cdot \alpha \).

[The theorem we used thrice - say \((PQR) \cdot \frac{RS}{QR} = (PRS)\) - is no more than adding metric to - see \text{EW}D1221\text{b} - \text{R} \parallel \text{S} \land \text{col.R.S.Q} \land \text{tri.R.Q.P} \Rightarrow \text{tri.R.S.P} ]

The theorem proved in this note is of no importance; it is recorded here because I don’t remember this proof technique from my school-days.

Nuenen, 22 December 1995

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