The formula for \( \sin(\alpha + \beta) \)

About circles we use the following lemma: Let arc \( AB \) of a circle with diameter \( d \) subtend at the centre an angle of \( 2\varphi \). Then

(i) at any point \( P \) on the remainder of the periphery it subtends an angle of \( \varphi \), and

(ii) the length of chord \( AB \) equals \( d \cdot \sin \varphi \).

[By choosing \( P = P' \) such that \( AP' \) is a diameter, we make \( \angle ABP' \) a right angle and thus see \( AB = d \cdot \sin \varphi \).] For the rest of this note we choose \( d = 1 \).
With the angles at A and B equal to $\alpha$ and $\beta$ respectively, we have according to our lemma

$BC = \sin \alpha$ \hspace{1em} $AC = \sin \beta$ \hspace{1em} $AB = \sin (\alpha + \beta)$

and now observe, with CD the altitude on AB

\[
\begin{align*}
\frac{\sin (\alpha + \beta)}{\sin \alpha} &= \frac{AB}{AD + DB} \\
&= AC \cos \alpha + BC \cos \beta \\
&= \sin \beta \cos \alpha + \sin \alpha \cos \beta
\end{align*}
\]

which establishes the addition formula for $\sin (\alpha + \beta)$ for $0 \leq \alpha, \beta \leq \pi/2$.

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