Mathematical induction's fixpoint

Let r be a left-founded relation, i.e. let

(0) $\langle \forall x :: [x \Rightarrow r;x] \Rightarrow [x \Rightarrow false] \rangle$;

an appeal to what is called "mathematical induction" means that a proof obligation of the form

(1) $[x \Rightarrow false]$

is replaced by the formally weaker obligation

(2) $[x \Rightarrow r;x]$ or $[x \wedge \neg(r;x) \Rightarrow false]$.

Proving (2) by mathematical induction leads to the (subsequently simplified) obligation

(3) $[x \wedge \neg(r;x) \Rightarrow r;(x \wedge \neg(r;x))]$

$\equiv$ {shunting}

$[x \Rightarrow r;x \vee r;(x \wedge \neg(r;x))]$

$\equiv$ {; monotonic in 2nd argument}

$[x \Rightarrow r;x]$,

i.e. (2)! The decision to prove by mathematical induction is idempotent. With thanks to the ETAC.

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