Eliminating cascading carries

In this note we describe an adder for numbers in the following decimal representation

\[(0) \quad n = \langle \Sigma i: 0 \leq i: d_i \cdot 10^i \rangle \]

\[(1) \quad -5 \leq d_i \leq 6 \quad \text{for all } i \]

All integers can be represented this way, but note that the representation is not necessarily unique: for instance, 26 can be represented in two decimals by \((2, 6)\) as well as by \((3, -4)\).

In our "adder", which can calculate \(\pm a \pm b\), the sum of two digits ranges from \(-12\) through \(+12\); its addition table represents these values as \(c \cdot 10 + s\) - from "carry" and "sum digit" - , with \(c, s\) satisfying

\[(2) \quad -1 \leq c \leq +1 \quad \text{and} \quad -4 \leq s \leq +5 \]

i.e., from \(-12\) through \(-5\), \(c = -1\),

from \(-4\) through \(+5\), \(c = 0\), and

from \(+6\) through \(+12\), \(c = +1\).

Because \((2)\) excludes for \(s\) the extreme digit values \(-5\) and \(+6\) and \(|c| \leq 1\), each
sum digit can absorb a carry from the right without generating a new one. In long parallel adders the problem of carry propagation has thus been eliminated; alternatively we can add from left to right with a "look-ahead" of only 1 position.

Remark If we so desire, we may replace (1) by the weaker, symmetric \(-6 \leq d_i \leq +6\). With (2) weakened accordingly, some freedom in the addition table is introduced. (End of Remark)

Please note that (i) the representation of zero is unique, and (ii) the sign of a non-zero value is determined by the sign of its most-significant non-zero digit.

The above, which was designed decades ago, was inspired by the implementation of the end-around carry in the serial adder of the ARMAC. Since the advent of systolic arrays it must look like something familiar.

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prof. dr Edsger W. Dijkstra
Department of Computer Sciences
The University of Texas at Austin
Austin, TX 78712 -1188
USA