Coxeter's rabbit

On p. 13 of his "Introduction to Geometry", H. S. M. Coxeter invites the reader to see (and to use spontaneously) that with \( s = (a+b+c)/2 \), \( abc \) equals

\[
0:\quad s(s-b)(s-c) + s(s-c)(s-a) + s(s-a)(s-b) - (s-a)(s-b)(s-c)
\]

\textbf{Proof}

\[
egin{align*}
(s-b)(s-c) + s(s-c)(s-a) & = \{ \text{algebra} \} \\
& = s(s-c)(2s-a-b) \\
& = \{ \text{definition of } s \} \\
& = s(s-c)c \\
\end{align*}
\]

\[
(1) \quad s(s-a)(s-b) - (s-a)(s-b)(s-c)
\]

\[
= \{ \text{algebra} \} \\
(2) \quad (s-a)(s-b)c
\]

Because both expressions (1) and (2) contain a factor \( c \), so does (0); for reasons of symmetry, (0) also contains factors \( a \) and \( b \), i.e. is a multiple of \( abc \). The coefficient equals 1 - as is trivially established with, say, \( a, b, c := 2, 2, 2 \) - and thus \( abc = (0) \) has been proved. \hspace{1cm} (End of Proof)

Nuenen, 14 April 2002

Prof. dr Edsger W. Dijkstra
Plataanstraat 5
5671 AA Nuenen
The Netherlands