Aliasing and Antialiasing

If the scene contains frequencies greater than the Nyquist Frequency, then we have an aliasing problem

Results of aliasing:

- Jaggies
- Moire
- Flickering small objects
- Sparkling highlights
- Temporal strobing

Preventing aliasing or antialiasing:

1. Analytically prefilter the signal
2. Uniform supersampling and resample
3. Nonuniform or stochastic sampling

Spectral Analysis / Fourier Transforms

Spectral representation treats the function as a weighted sum of sines and cosines

Each function has two representations

- Spatial (time) domain – normal representation
- Frequency domain – spectral representation

The *Fourier transform* converts between the spatial and frequency domain

Spatial Domain	$\Longrightarrow$	$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$	$\Longrightarrow$	Frequency Domain
	$\Longleftarrow$	$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega x} d\omega$	$\Longleftarrow$	

Convolution

Definition

$$h(x) = f \otimes g = \int f(x')g(x - x')dx'$$

Convolution Theorem: Multiplication in the frequency domain is equivalent to convolution in the space domain

$$f \otimes g \longleftrightarrow F \times G$$

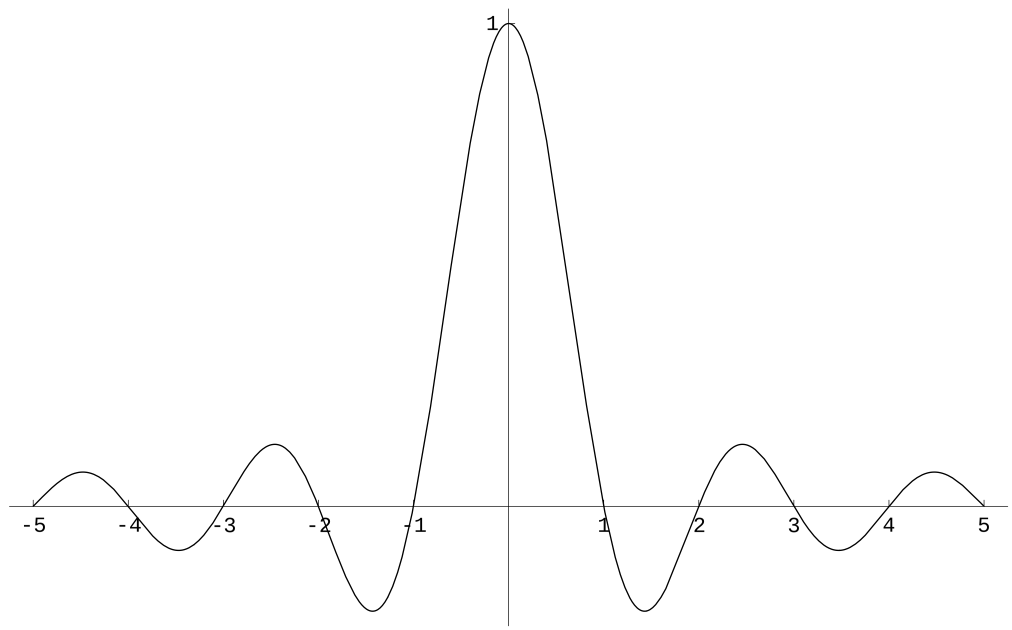
Symmetric Theorem: Multiplication in the space domain is equivalent to convolution in the frequency domain

$$f \times g \longleftrightarrow F \otimes G$$

1-D Sinc Function

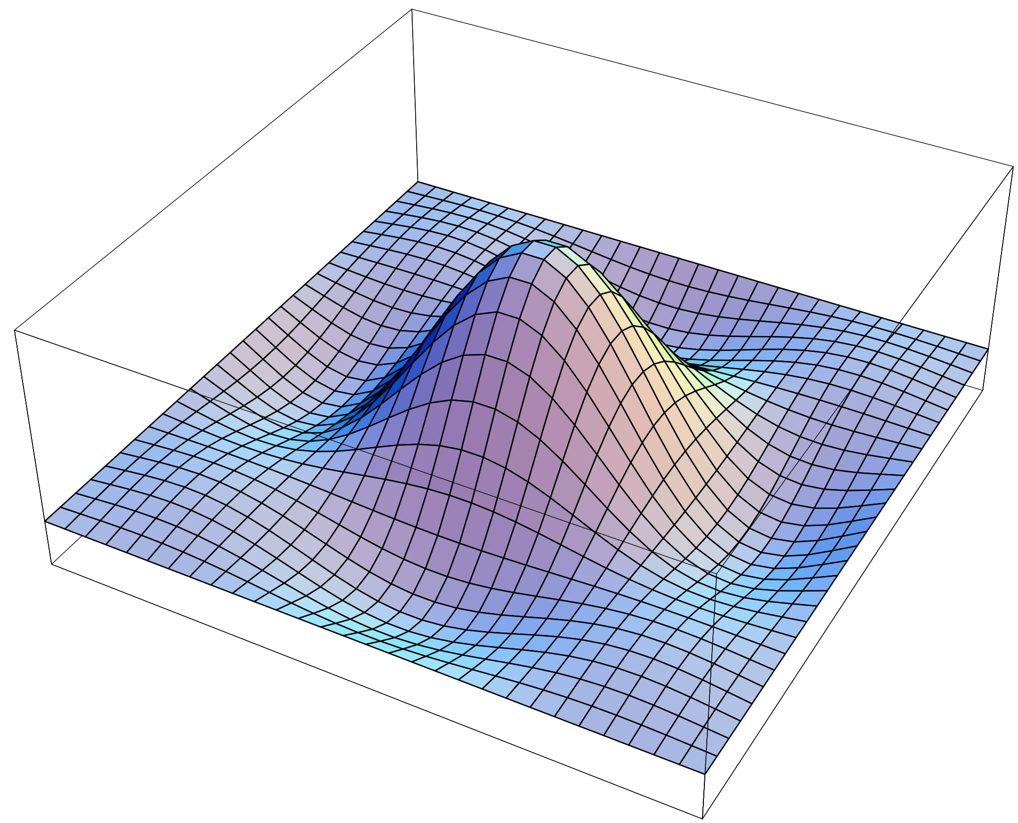
$$\mathsf{sinc}\, x = \frac{\sin \pi x}{\pi x}$$

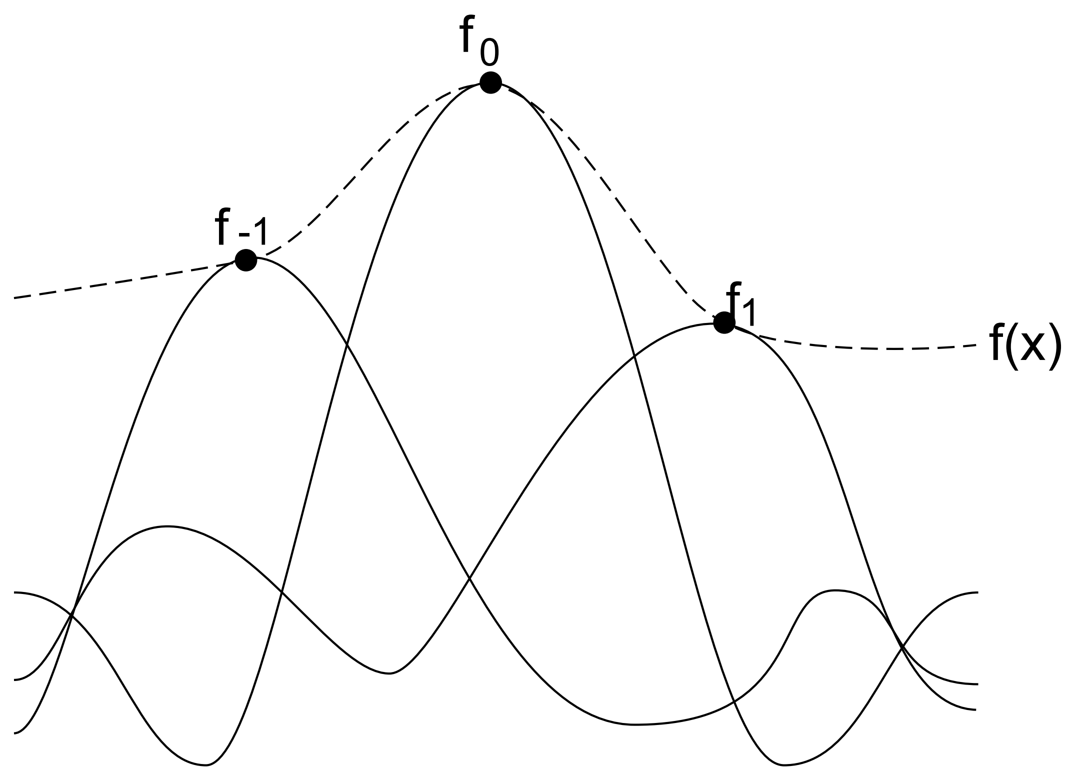
$$\mathsf{sinc}\, 0 = 1$$



2-D Sinc Function

$$\mathsf{sinc}\,(x, y) = \mathsf{sinc}\,(x)\ \mathsf{sinc}\,(y)$$




f_0
f_{-1}
f_1
f(x)

Nyquist Sampling Theorem

A signal can be reconstructed from its samples without loss of information, if the original signal has no frequencies above 1/2 the sampling frequency

For a given bandlimited function, the rate at which it must be sampled is called the Nyquist Frequency

Nyquist Sampling Theorem (Part 2)

We can reconstruct a continuous function $f(x)$ from its samples $\{f_i\}$ by the formula

$$f(x, y) = \sum_{i=-\infty}^{\infty} f_i \,\mathsf{sinc}\,(x - x_i).$$

The two-dimensional version of the reconstruction formula for a function $f(x, y)$ with ideal samples $\{f_{ij}\}$ is

$$f(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} f_{ij} \,\mathsf{sinc}\,(x - x_i) \,\mathsf{sinc}\,(y - y_i).$$

Ideal Reconstruction

Ideally, use a perfect low-pass filter – the sinc function – to bandlimit the sampled signal and thus remove all copies of the spectra introduced by sampling

Unfortunately,

- The sinc has infinite extent and we must use simpler filters with finite extents. Physical processes in particular do not reconstruct with sincs
- The sinc may introduce ringing which are perceptually objectionable

Antialiasing by Prefiltering

Ideally, low-pass with a perfect filter (a sinc function) to bandlimit the function to the Nyquist sampling rate.

Unfortunately, the sinc has infinite extent and we must use simpler filters (like a box filter, or area average).

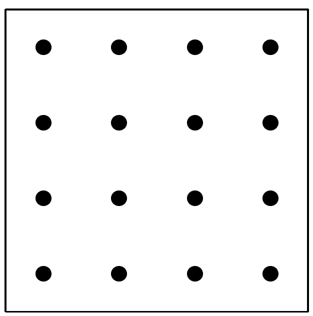
Practically:

- Constant colored polygonal fragments doable
- Complex environments not doable

Uniform Supersampling

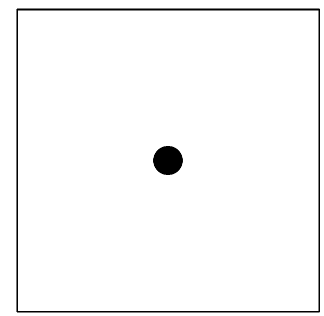
Increasing the sampling rate moves each copy of the spectra further apart, potentially reducing the overlap and thus aliasing

Resulting samples must be resampled (filtered) to image sampling rate



Samples

$$\text{Pixel} = \sum_s w_s \text{Sample}_s$$



Samples

Non-uniform Sampling

Uniform sampling

- The spectrum of uniformly spaced samples is also a set of uniformly spaced spikes
- Multiplying the signal by the sampling pattern corresponds to placing a copy of the spectrum at each spike (in frequency space)
- Aliases are coherent, and very noticable

Non-uniform sampling

- Samples at non-uniform locations have a different spectrum; a single spike plus noise
- Sampling a signal in this way converts aliases into broadband noise
- Noise is incoherent, and much less objectionable