Clipping and Intersection

Clipping: Remove points, line segments, polygons outside a region of interest.

- Need to discard everything that's outside of our window.

Point clipping: Remove points outside window.

- A point is either entirely inside the region or not.

Line clipping: Remove portion of line segment outside window.

- Line segments can straddle the region boundary.
- The Liang-Barsky algorithm efficiently clips line segments against a halfspace.
- Halfspaces can be combined to bound a convex region.
- Use *outcodes* to organize combination of halfspaces.
- Can use some of the ideas in Liang-Barsky to clip points.

Polygon clipping: Remove portion of polygon outside window.

- Polygons can straddle the region boundary.
- Concave polygons can be broken into convex.
- Sutherland-Hodgemen algorithm also deals with all cases efficiently.
- Built upon efficient line segment clipping.

Parametric representation of line:

$$P(t) = (1 - t)P_0 + tP_1$$

or equivalently:

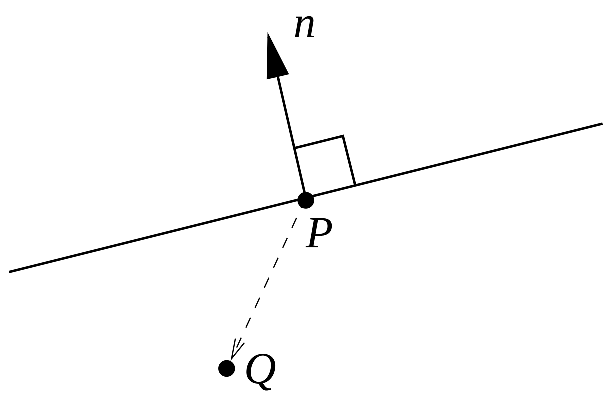
$$P(t) = P_0 + t(P_1 - P_0)$$

- P_0 and P_1 are non-coincident points.
- For $t \in \mathbf{R}$, $P(t)$ defines an infinite line.
- For $t \in [0, 1]$, $P(t)$ defines a line segment from P_0 to P_1.
- Good for generating points on a line.
- Not so good for testing if a given point is on a line.

Implicit representation of line:

$$\ell(Q) = (Q - P) \cdot \vec{n}$$

- P is a point on the line.
- $\vec{n}$ is a vector perpendicular to the line.
- $\ell(Q)$ gives us the signed distance from any point Q to the line.
- The sign of $\ell(Q)$ tells us if Q is on the left or right of the line, relative to the direction of $\vec{n}$.
- If $\ell(Q)$ is zero, then Q is on the line.
- Use same form for the implicit representation of a halfspace.

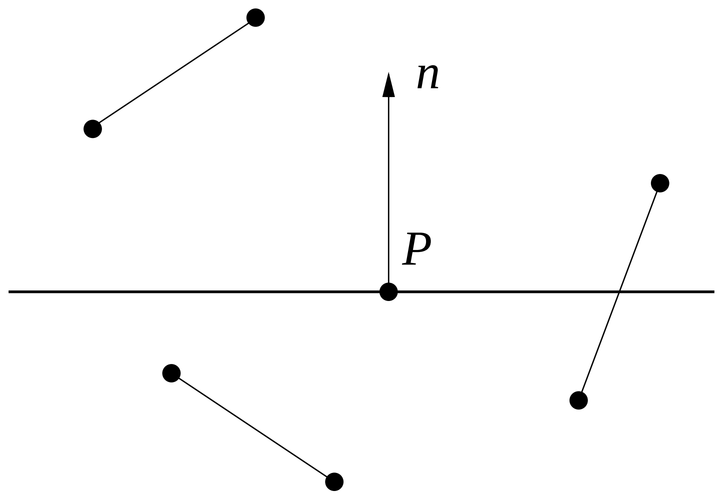


Clipping a point to a halfspace:

- Represent a window edge implicitly...
- Use the implicit form of a line to classify a point Q.
- Must choose a convention for the normal: point to the *inside*.
- Check the sign of $\ell(Q)$:
  - If $\ell(Q) > 0$, the Q is inside.
  - Otherwise clip (discard) Q; it is on, or outside.

Clipping a line segment to a halfspace: There are three cases:

- The line segment is entirely inside.
- The line segment is entirely outside.
- The line segment is partially inside and partially outside.



Do the easy stuff first: We can devise easy (and fast!) tests for the first two cases:

- $(P_0 - P) \cdot \vec{n} < 0$ AND $(P_1 - P) \cdot \vec{n} < 0 \Longrightarrow$ Outside
- $(P_0 - P) \cdot \vec{n} > 0$ AND $(P_1 - P) \cdot \vec{n} > 0 \Longrightarrow$ Inside

We will also need to decide whether "on the boundary" is inside or outside.

Trivial tests are important in computer graphics:

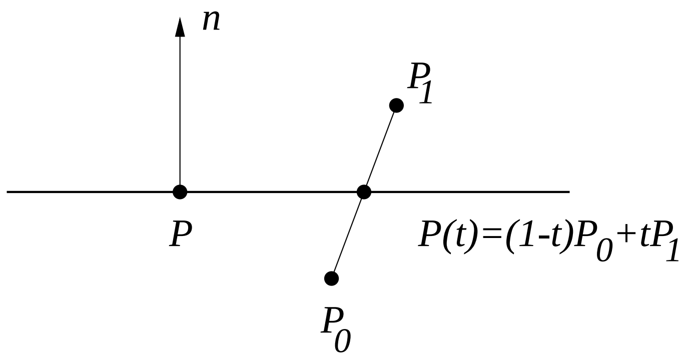
- Particularly if the trivial case is the most common one.
- Particularly if we can reuse the computation for the non-trivial case.

Do the hard stuff only if we have to: If line segment partially inside and partially outside, need to clip it:

- Represent the segment from P_0 to P_1 in parametric form:

$$P(t) = (1 - t)P_0 + tP_1 = P_0 + t(P_1 - P_0)$$

- When $t = 0$, $P(t) = P_0$. When $t = 1$, $P(t) = P_1$.
- We now have the following:



- We want t such that $P(t)$ is on ℓ:

$$\begin{aligned}(P(t)-P)\cdot\vec{n} &= (P_0+t(P_1-P_0)-P)\cdot\vec{n}\\ &= (P_0-P)\cdot\vec{n}+t(P_1-P_0)\cdot\vec{n}\\ &= 0\end{aligned}$$

- Solving for t gives us

$$t=\frac{(P_0-P)\cdot\vec{n}}{(P_0-P_1)\cdot\vec{n}}$$

- NOTE: The values we use for our simple test can be used to compute t:

$$t=\frac{(P_0-P)\cdot\vec{n}}{(P_0-P)\cdot\vec{n}-(P_1-P)\cdot\vec{n}}$$

Clipping a line segment to a window: Just clip to each of four halfspaces.

Pseudo-code:

```
given:  P, n defining a window edge
for each edge (A,B) = (P0,P1)
    wecA = (A-P) . n
    wecB = (B-P) . n
    if ( wecA < 0 AND wecB < 0 ) then reject
    if ( wecA >= 0 AND wecB >= 0 ) then next
    t = wecA / (wecA - wecB)
    if (wecA < 0 ) then
        A = A + t*(B-A)
    else
        B = B + t*(B-A)
    endif
endfor
```

NOTE:

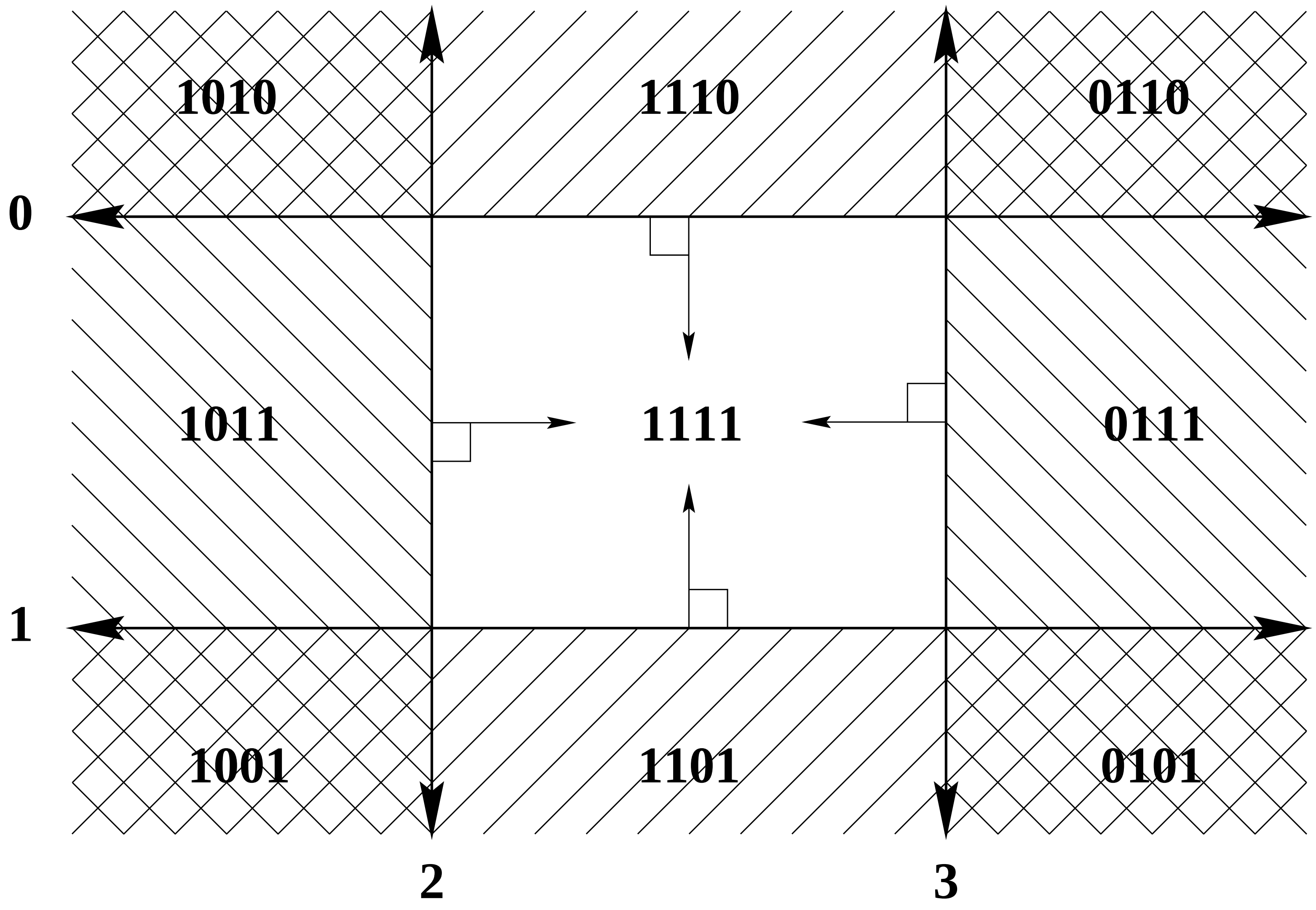
- Liang-Barsky Algorithm lets us clip lines to arbitrary convex windows.

- Optimizations can be made for the special case of horizontal and vertical window edges.

Q: Can we short-circuit evaluation of a clip on one edge if we know line segment is out on another?

A: Yes. Use *outcodes*.

- Do all trivial tests first.
- Combine results using Boolean operations to determine
  - Trivial accept on window: all edges have trivial accept.
  - Trivial reject on window: any edge has trivial reject.
- Do Boolean AND, OR operations on bits in a packed integer.
- Do full clip only if no trivial accept/reject on *window*.


1010
1110
0110
0
1011
1111
0111
1
1001
1101
0101
2
3

Line-clip Algorithm generalizes to 3D:

- Half-space now lies on one side of a *plane*.
- The implicit formula for a plane in 3D is the same as that for a line in 2D.
- The parametric formula for the line to be clipped is unchanged.

3D Clipping

- When do we clip in 3D? We should clip to the near plane *before* we project. Otherwise, we might map z to 0 and the x/z and y/z are undefined.
- We could clip to all 6 sides of the truncated viewing pyramid. but the plane equations are simpler if we clip after projection, because all sides of volume are parallel to coordinate plane.
- Clipping to a plane in 3D is identical to clipping to a line in 2D.
- We can also clip in homogeneous coordinates.

Polygon Clipping

Polygon Clipping (Sutherland-Hodgeman):

- Window must be a convex polygon.
- Polygon to be clipped can be convex or not.

Approach:

- Polygon to be clipped is given as $v_1, \ldots, v_n$
- Each polygon edge is a pair $[v_i, v_{i+1}] i = 1, \ldots, n$
  - Don't forget wraparound; $[v_n, v_1]$ is also an edge
- Process all polygon edges in succession against a window edge
  - Polygon in – polygon out
  - $v_1, \ldots, v_n \rightarrow w_1, \ldots, w_m$
- Repeat on resulting polygon with next sequential window edge.

Contrast with Line Clipping

- Line clipping:
  - Use outcodes to check all window edges before any clip
  - Clip only against possible intersecting window edges
  - Deal with window edges in any order
  - Deal with line segment endpoints in either order
- Polygon clipping:
  - Each window edge must be used
  - Polygon edges must be handled in sequence
  - Polygon edge endpoints have a given order
  - Stripped-down line-segment/window-edge clip is a subtask

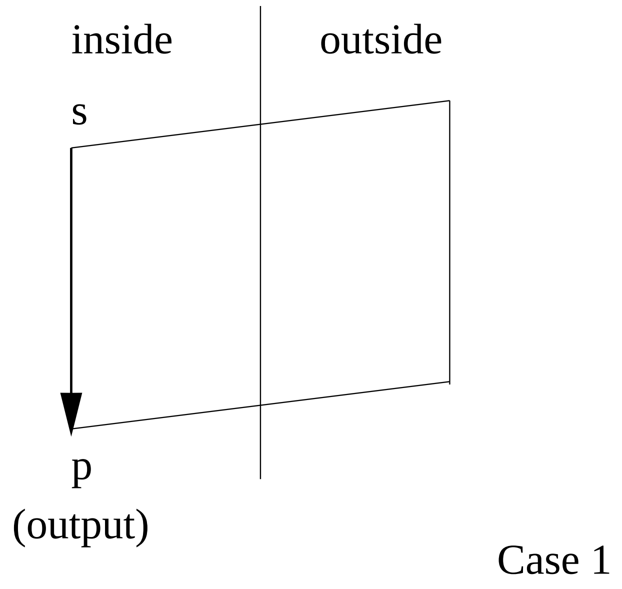
There are four cases to consider.

Four Cases

- $s = v_i$ is the polygon edge starting vertex
- $p = v_{i+1}$ is the polygon edge ending vertex
- i is a polygon-edge/window-edge intersection point
- w_j is the next polygon vertex to be output

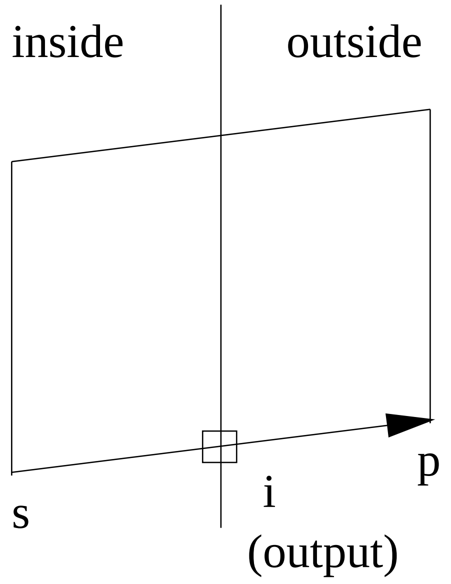
Case 1: Polygon edge is entirely inside the window edge

- p is next vertex of resulting polygon
- $p \rightarrow w_j$ and $j + 1 \rightarrow j$


inside
outside
s
p
(output)
Case 1

Case 2: Polygon edge crosses window edge going out

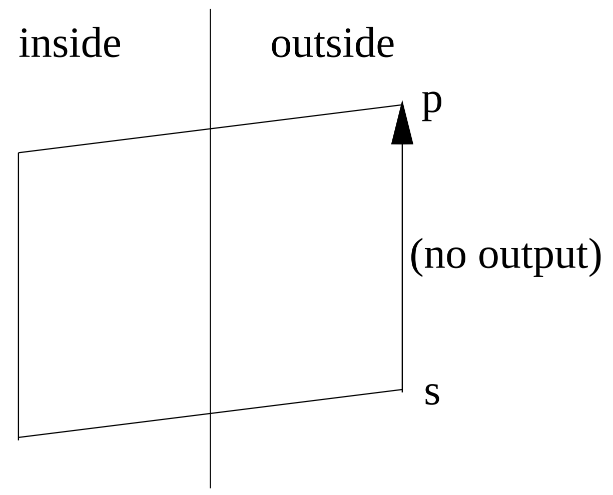
- Intersection point i is next vertex of resulting polygon
- $i \rightarrow w_j$ and $j + 1 \rightarrow j$



Case 2

Case 3: Polygon edge is entirely outside the window edge

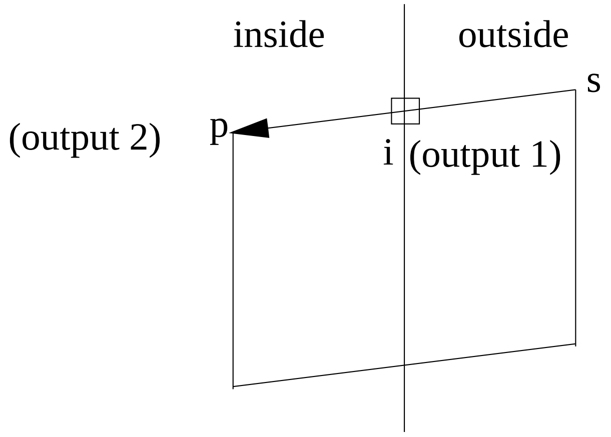
- No output



Case 3

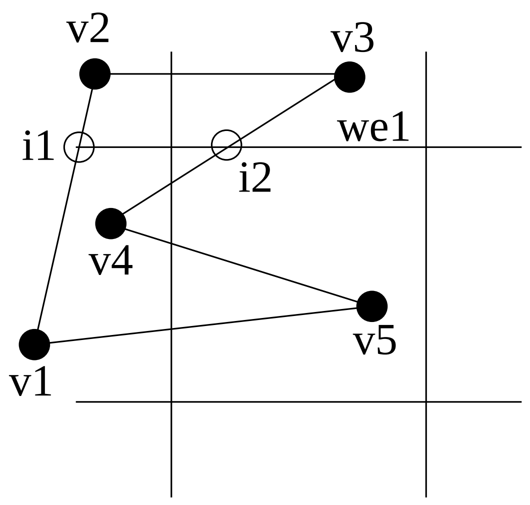
Case 4: Polygon edge crosses window edge going in

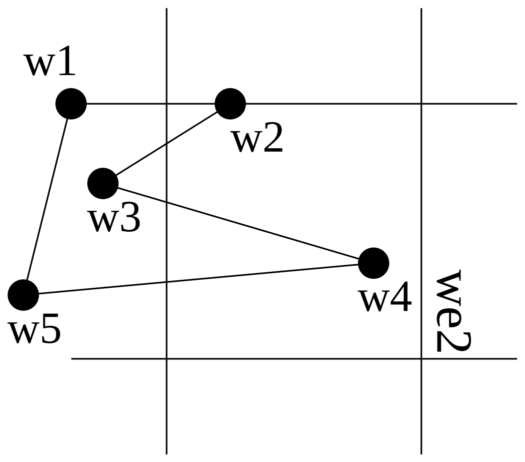
- Intersection point i and p are next two vertices of resulting polygon
- $i \rightarrow w_j$ and $p \rightarrow w_{j+1}$ and $j+2 \rightarrow j$

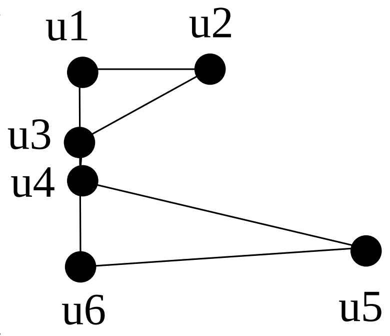


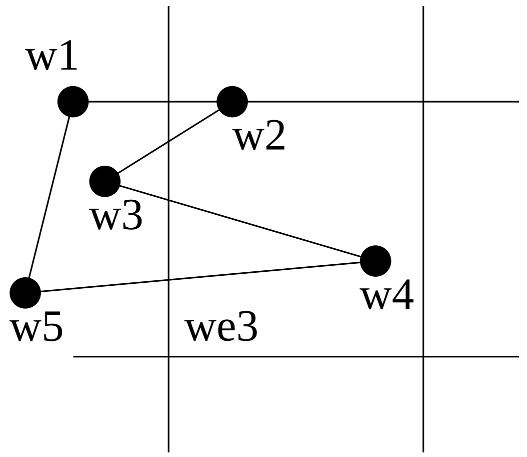
Case 4

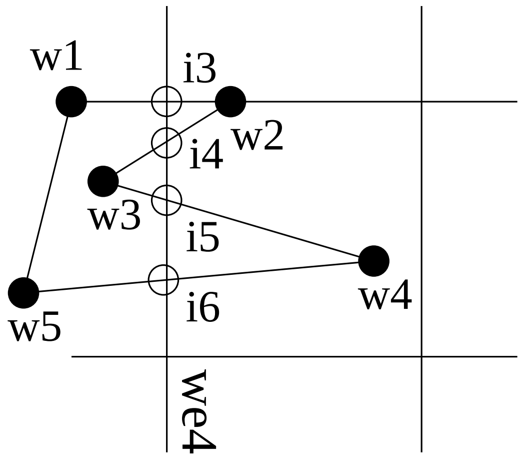
An Example with a Non-convex Polygon


v2
v3
i1
i2
we1
v4
v5
v1


w1
w2
w3
w4
w5
we2


u1
u2
u3
u4
u6
u5


w1
w2
w3
w4
w5
we3


w1
i3
w2
i4
w3
i5
w4
w5
i6
we4

Ray-Polyhedron Intersection

(This algorithm is a variation/extension of the Liang-Barsky line segment clipping algorithm)

The convex polyhedron is defined as the intersection of a collection of halfspaces in 3-space. Represent the ray parametrically as $P(t) = P + t\vec{u}$, for scalar $t > 0$, point P and a unit vector $\vec{(u)}$. Let H_1, H_2, . . . , H_k denote the halfspaces defining the polyhedron. Compute the intersection of the ray with each halfspace in turn. The final result will be the intersection of the ray with the entire polyhedron.

An important property of convex bodies (of any variety) is that a line intersects a convex body in at most one line segment. Thus the resulting intersection of the ray with the polyhedron can be specified entirely by an interval of scalars $[t_0, t_1]$. The parametric representation of the intersection is defined by the line segment Initially, let this interval be $[0, \infty]$.

(For a line-segment intersection with a polyhedron, the only change is that an initial value of t_1 is set as the endpoint of the segment rather than ∞).

Suppose we have already performed the intersection with some number of the halfspaces. It might be that the intersection is already empty. This will be reflected by the fact that

$t_0 > t_1$. When this is so, we may terminate the algorithm at any time. Otherwise, let $h = (a, b, c, d)$ be the coefficients of the current halfspace.

We want to know the value of t (if any) at which the ray intersects the plane. Plugging in the representation of the ray into the halfspace inequality we have

$$a(p_x + t\vec{u}_x) + b(p_y + \vec{u}_y) + c(p_z + t\vec{u}_z) + d \leq 0,$$

which after some manipulations is

$$t(a\vec{u}_x + b\vec{u}_y + c\vec{u}_z) \leq -(ap_x + bp_y + cp_z + d)$$

If P and $\vec{u}$ are given in homogeneous coordinates, this can be written as

$$t(H \cdot \vec{u}) \leq -(H \cdot P)$$

This is not really a legitimate geometric expression (since dot product should only be applied between vectors). Actually the halfspace H should be thought of as a special geometric object, a sort of *generalized normal vector*. For example, when transformations are applied, normal vectors should be multiplied by the inverse transpose matrix to maintain orthogonality.

We consider three cases.

$(H \cdot \vec{u}) > 0$: in this case we have the constraint

$$t \leq \frac{-(H \cdot P)}{(H \cdot \vec{u})}.$$

Let t^* denote the right-hand side of this inequality. We trim the high-end of the intersection interval to $[t_0, \min(t_1, t^*)]$.

$(H \cdot \vec{u}) < 0$ in this case we have

$$t \geq \frac{-(H \cdot P)}{(H \cdot \vec{u})}.$$

Let t^* denote the right hand side of the inequality. In this case, we trim the low-end of the intersection interval to $[t_0, \min(t_1, t^*)]$.

$(H \cdot \vec{u}) = 0$: In this case the ray is parallel to the plane, either entirely above or below or on. We check the origin. If $(H \cdot P) <= 0$ then the origin lies in (or on the boundary of) the halfspace, and so we leave the current interval unchanged. Otherwise, the origin lies outside the halfspace, and the intersection is empty. To model this we can set t_1 to any negative value, e.g., -1.

After we repeat this on each face of the polyhedron, we have the following possibilities:

$t_1 < t_0$: In this case the ray does not intersect the polyhedron.

$0 = t_0 = t_1$: In this case, the origin is within the polyhedron. If $t_1 = \infty$, then the polyhedron must be ungrounded (e.g. like a cone) and there is no intersection. Otherwise, the first intersection point is the point $P + t_1\vec{u}$.

$0 < t_0 \leq t_1$: In this case, the origin is outside the polyhedron, and the first intersection is at $P + t_0\vec{u}$.

Reading Assignment and News

Chapter 8 pages 374 - 387, of Recommended Text.

Please also track the News section of the Course Web Pages for the most recent Announcements related to this course.

(http://www.cs.utexas.edu/users/bajaj/graphics23/cs354/)