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EVENT: Start with the library "proveall" using the compiled version.

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;            Sums of the Fibonacci Numbers

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; This file naturally belongs over in the cowles subdirectory
; rather than in the basic subdirectory, but we put it here because
; it depends upon proveall, proveall is long, and we choose not
; to put operating system/directory syntax information into these files.

; This event list contains several interesting theorems
; about the Fibonacci numbers such as
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; The SUM, from k=0 to k=n, of fib( k ) = fib( n+2 ) - 1.
;
; The SUM, from k=0 to k=n, of fib( 2*k+1 ) = fib( 2*n+2 ).

; Here fib( i ) is the ith Fibonacci number.
```

DEFINITION:
fib (n)
$=$ **if** $n \simeq 0$ **then** 0
elseif $n = 1$ **then** 1
else fib $(n - 1)$ + fib $((n - 1) - 1)$ **endif**

DEFINITION:
nbr-calls-fib (n)
$=$ **if** $n \simeq 0$ **then** 1
elseif $n = 1$ **then** 1
else 1 + (nbr-calls-fib $(n - 1)$ + nbr-calls-fib $((n - 1) - 1)$) **endif**

DEFINITION:
nbr-plus-fib (n)
$=$ **if** $n \simeq 0$ **then** 0
elseif $n = 1$ **then** 0
else 1 + (nbr-plus-fib $(n - 1)$ + nbr-plus-fib $((n - 1) - 1)$) **endif**

THEOREM: nbr-calls-fib&fib
(1 + nbr-calls-fib (n)) = (2 $*$ fib $(1 + n)$)

THEOREM: nbr-calls-fib=fib
nbr-calls-fib (n) = ((2 $*$ fib $(1 + n)$) $-$ 1)

THEOREM: nbr-plus-fib&fib
(1 + nbr-plus-fib (n)) = fib $(1 + n)$

THEOREM: nbr-plus-fib=fib
nbr-plus-fib (n) = (fib $(1 + n)$ $-$ 1)

DEFINITION:
sum-fib<k> (n)
$=$ **if** $n \simeq 0$ **then** 0
else sum-fib<k> $(n - 1)$ + fib (n) **endif**

THEOREM: sum-fib<k>&fib
(1 + sum-fib<k> (n)) = fib $(1 + (1 + n))$

THEOREM: sum-fib<k>=fib
sum-fib<k> (n) = (fib $(1 + (1 + n))$ $-$ 1)

Theorem: sum-fib<k>=nbr-plus-fib
sum-fib<k> (n) = nbr-plus-fib (1 + n)

Definition:
sum-fib<2k> (n)
= **if** $n \simeq 0$ **then** 0
else sum-fib<2k> (n − 1) + fib (2 ∗ n) **endif**

Theorem: sum-fib<2k>&fib
(1 + sum-fib<2k> (n)) = fib (1 + (2 ∗ n))

Theorem: sum-fib<2k>=fib
sum-fib<2k> (n) = (fib (1 + (2 ∗ n)) − 1)

Definition:
sum-fib<2k+1> (n)
= **if** $n \simeq 0$ **then** 1
else sum-fib<2k+1> (n − 1) + fib (1 + (2 ∗ n)) **endif**

Theorem: sum-fib<2k+1>=fib
sum-fib<2k+1> (n) = fib (1 + (1 + (2 ∗ n)))

Definition:
sum-fib<3k> (n)
= **if** $n \simeq 0$ **then** 0
else sum-fib<3k> (n − 1) + fib (3 ∗ n) **endif**

Theorem: sum-fib<3k>&fib
(1 + (2 ∗ sum-fib<3k> (n))) = fib (1 + (1 + (3 ∗ n)))

Theorem: sum-fib<3k>&fib-1
(2 ∗ sum-fib<3k> (n)) = (fib (1 + (1 + (3 ∗ n))) − 1)

Theorem: sum-fib<3k>=fib
sum-fib<3k> (n) = ((fib (1 + (1 + (3 ∗ n))) − 1) ÷ 2)

Definition:
sum-fib<3k+1> (n)
= **if** $n \simeq 0$ **then** 1
else sum-fib<3k+1> (n − 1) + fib (1 + (3 ∗ n)) **endif**

Theorem: sum-fib<3k+1>&fib
(2 ∗ sum-fib<3k+1> (n)) = fib (1 + (1 + (1 + (3 ∗ n))))

Theorem: sum-fib<3k+1>=fib
sum-fib<3k+1> (n) = (fib (1 + (1 + (1 + (3 ∗ n)))) ÷ 2)

DEFINITION:

$$\text{sum-fib<3k+2>}\,(n) = \textbf{if } n \simeq \mathtt{0} \textbf{ then } \mathtt{1} \textbf{ else } \text{sum-fib<3k+2>}\,(n-1) + \text{fib}\,(1 + (1 + (\mathtt{3} * n))) \textbf{ endif}$$

THEOREM: sum-fib<3k+2>&fib

$$(1 + (\mathtt{2} * \text{sum-fib<3k+2>}\,(n))) = \text{fib}\,(1 + (1 + (1 + (1 + (\mathtt{3} * n)))))$$

THEOREM: sum-fib<3k+2>&fib-1

$$(\mathtt{2} * \text{sum-fib<3k+2>}\,(n)) = (\text{fib}\,(1 + (1 + (1 + (1 + (\mathtt{3} * n))))) - 1)$$

THEOREM: sum-fib<3k+2>=fib

$$\text{sum-fib<3k+2>}\,(n) = ((\text{fib}\,(1 + (1 + (1 + (1 + (\mathtt{3} * n))))) - 1) \div \mathtt{2})$$

THEOREM: fib<2n+1>=fib<n>&fib<2n+2>=fib<n>

$$\begin{aligned}
&(\text{fib}\,(1 + (\mathtt{2} * n)) \\
&\quad = \quad ((\text{fib}\,(n) * \text{fib}\,(n)) + (\text{fib}\,(1+n) * \text{fib}\,(1+n)))) \\
&\land \quad (\text{fib}\,(1 + (1 + (\mathtt{2} * n))) \\
&\qquad = \quad ((\text{fib}\,(n) * \text{fib}\,(1+n)) \\
&\qquad\qquad + \quad (\text{fib}\,(1+n) * \text{fib}\,(1 + (1+n)))))
\end{aligned}$$

THEOREM: fib<2n+1>=fib<n>

$$\text{fib}\,(1 + (\mathtt{2} * n)) = ((\text{fib}\,(n) * \text{fib}\,(n)) + (\text{fib}\,(1+n) * \text{fib}\,(1+n)))$$

THEOREM: fib<2n+2>=fib<n>

$$\text{fib}\,(1 + (1 + (\mathtt{2} * n))) = ((\text{fib}\,(n) * \text{fib}\,(1+n)) + (\text{fib}\,(1+n) * \text{fib}\,(1 + (1+n))))$$

DEFINITION:

$$\text{sum-fib<4k>}\,(n) = \textbf{if } n \simeq \mathtt{0} \textbf{ then } \mathtt{0} \textbf{ else } \text{sum-fib<4k>}\,(n-1) + \text{fib}\,(\mathtt{4} * n) \textbf{ endif}$$

DEFINITION:

$$\text{sum-fib<4k+1>}\,(n) = \textbf{if } n \simeq \mathtt{0} \textbf{ then } \mathtt{1} \textbf{ else } \text{sum-fib<4k+1>}\,(n-1) + \text{fib}\,(1 + (\mathtt{4} * n)) \textbf{ endif}$$

DEFINITION:

$$\text{sum-fib<4k+2>}\,(n) = \textbf{if } n \simeq \mathtt{0} \textbf{ then } \mathtt{1} \textbf{ else } \text{sum-fib<4k+2>}\,(n-1) + \text{fib}\,(1 + (1 + (\mathtt{4} * n))) \textbf{ endif}$$

DEFINITION:

$$\text{sum-fib<4k+3>}\,(n) = \textbf{if } n \simeq \mathtt{0} \textbf{ then } \mathtt{2} \textbf{ else } \text{sum-fib<4k+3>}\,(n-1) + \text{fib}\,(1 + (1 + (1 + (\mathtt{4} * n)))) \textbf{ endif}$$

THEOREM: times-4=times-2-2

$$(4 * n) = (2 * 2 * n)$$

THEOREM: fib<4n+1>=fib<2n>

$$\begin{aligned} & \mathrm{fib}\,(1 + (4 * n)) \\ = \quad & ((\mathrm{fib}\,(2 * n) * \mathrm{fib}\,(2 * n)) \\ & \quad + \quad (\mathrm{fib}\,(1 + (2 * n)) * \mathrm{fib}\,(1 + (2 * n)))) \end{aligned}$$

THEOREM: sum-fib<4k+1>=fib<2n>

$$\text{sum-fib<4k+1>}\,(n) = (\mathrm{fib}\,(1 + (2 * n)) * \mathrm{fib}\,(1 + (1 + (2 * n))))$$

THEOREM: fib<4n+2>=fib<2n>

$$\begin{aligned} & \mathrm{fib}\,(1 + (1 + (4 * n))) \\ = \quad & ((\mathrm{fib}\,(2 * n) * \mathrm{fib}\,(1 + (2 * n))) \\ & \quad + \quad (\mathrm{fib}\,(1 + (2 * n)) * \mathrm{fib}\,(1 + (1 + (2 * n))))) \end{aligned}$$

THEOREM: sum-fib<4k+2>=fib<2n>

$$\begin{aligned} & \text{sum-fib<4k+2>}\,(n) \\ = \quad & (\mathrm{fib}\,(1 + (1 + (2 * n))) * \mathrm{fib}\,(1 + (1 + (2 * n)))) \end{aligned}$$

THEOREM: fib<4n+3>=fib<2n>

$$\begin{aligned} & \mathrm{fib}\,(1 + (1 + (1 + (4 * n)))) \\ = \quad & ((\mathrm{fib}\,(1 + (2 * n)) * \mathrm{fib}\,(1 + (2 * n))) \\ & \quad + \quad (\mathrm{fib}\,(1 + (1 + (2 * n))) * \mathrm{fib}\,(1 + (1 + (2 * n))))) \end{aligned}$$

THEOREM: sum-fib<4k+3>=fib<2n>

$$\begin{aligned} & \text{sum-fib<4k+3>}\,(n) \\ = \quad & (\mathrm{fib}\,(1 + (1 + (1 + (2 * n)))) * \mathrm{fib}\,(1 + (1 + (2 * n)))) \end{aligned}$$

THEOREM: fib<2n>=fib<n>

$$\begin{aligned} & (0 < n) \\ \to \quad & (\mathrm{fib}\,(2 * n) \\ & \quad = \quad ((\mathrm{fib}\,(n - 1) * \mathrm{fib}\,(n)) + (\mathrm{fib}\,(n) * \mathrm{fib}\,(1 + n)))) \end{aligned}$$

THEOREM: fib<4n>=fib<2n>

$$\begin{aligned} & (0 < n) \\ \to \quad & (\mathrm{fib}\,(4 * n) \\ & \quad = \quad ((\mathrm{fib}\,((2 * n) - 1) * \mathrm{fib}\,(2 * n)) \\ & \qquad + \quad (\mathrm{fib}\,(2 * n) * \mathrm{fib}\,(1 + (2 * n))))) \end{aligned}$$

THEOREM: sum-fib<4k>=fib<2n>

$$\text{sum-fib<4k>}\,(n) = (\mathrm{fib}\,(2 * n) * \mathrm{fib}\,(1 + (1 + (2 * n))))$$

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