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; Work of David Russinoff.

EVENT: Start with the library "rsa" using the compiled version.

EVENT: For efficiency, compile those definitions not yet compiled.

DEFINITION:

inverse(j, p)

= **if** $p = 2$ **then** $j \bmod 2$
 else $\exp(j, p - 2) \bmod p$ **endif**

THEOREM: inverse-inverts-lemma

$(p \neq 0) \rightarrow (((\text{inverse}(j, p) * j) \bmod p) = (\exp(j, p - 1) \bmod p))$

THEOREM: inverse-inverts

$(\text{prime}(p) \wedge ((j \bmod p) \neq 0)) \rightarrow (((\text{inverse}(j, p) * j) \bmod p) = 1)$

THEOREM: inverse-is-unique

$$(\text{prime}(p) \wedge (1 = ((m * x) \bmod p))) \rightarrow (\text{inverse}(m, p) = (x \bmod p))$$

THEOREM: s-p-i-i-lemma1

$$\begin{aligned} & ((n \not\equiv 0) \wedge (n \neq 1)) \\ \rightarrow & (((n - 1) * (n - 1)) = (1 + (n * ((n - 1) - 1)))) \end{aligned}$$

THEOREM: s-p-i-i-lemma2

$$((n \not\equiv 0) \wedge (n \neq 1)) \rightarrow (((n - 1) * (n - 1)) \bmod n = 1)$$

THEOREM: subl-p-is-involution

$$\text{prime}(p) \rightarrow (\text{inverse}(p - 1, p) = (p - 1))$$

THEOREM: n-o-i-lemma1

$$((x * x) - 1) = ((1 + x) * (x - 1))$$

THEOREM: n-o-i-lemma2

$$\begin{aligned} & (\text{prime}(p) \wedge (((j * j) - 1) \bmod p = 0)) \\ \rightarrow & (((1 + j) \bmod p = 0) \vee ((j - 1) \bmod p = 0)) \end{aligned}$$

THEOREM: n-o-i-lemma3

$$((a \not\equiv 1) \wedge ((a \bmod p) = 1)) \rightarrow (((a - 1) \bmod p) = 0)$$

THEOREM: n-o-i-lemma4

$$\begin{aligned} & (\text{prime}(p) \wedge ((j \bmod p) \neq 0) \wedge (\text{inverse}(j, p) = j)) \\ \rightarrow & (((1 + j) \bmod p = 0) \vee ((j - 1) \bmod p = 0)) \end{aligned}$$

THEOREM: no-other-involutions

$$(\text{prime}(p) \wedge (j < (p - 1)) \wedge (1 < j)) \rightarrow (\text{inverse}(j, p) \neq j)$$

THEOREM: i-o-i-lemma

$$(((p - 2) * (p - 2)) - 1) = ((p - 3) * (p - 1))$$

THEOREM: inverse-of-inverse

$$\begin{aligned} & (\text{prime}(p) \wedge ((j \bmod p) \neq 0)) \\ \rightarrow & (\text{inverse}(\text{inverse}(j, p), p) = (j \bmod p)) \end{aligned}$$

THEOREM: n-z-i-lemma

$$((i \simeq 0) \wedge (1 < p)) \rightarrow (\text{inverse}(i, p) = 0)$$

THEOREM: non-zero-p-inverse

$$(\text{prime}(p) \wedge ((j \bmod p) \neq 0)) \rightarrow (\text{inverse}(j, p) \not\equiv 0)$$

THEOREM: b-i-lemma2

$$\begin{aligned} & (\text{prime}(p) \wedge ((j \bmod p) \neq 0) \wedge (\text{inverse}(j, p) = (p - 1))) \\ \rightarrow & ((j \bmod p) = (p - 1)) \end{aligned}$$

THEOREM: b-i-lemma1

$$(1 < p) \rightarrow (\text{inverse}(j, p) \leq (p - 1))$$

THEOREM: bounded-inverse

$$(\text{prime}(p) \wedge (j < (p - 1))) \rightarrow (\text{inverse}(j, p) < (p - 1))$$

DEFINITION:

inverse-list(i, p)

= **if** $i \simeq 0$ **then** **nil**
 elseif $i = 1$ **then** cons(**1**, **nil**)
 elseif $i \in \text{inverse-list}(i - 1, p)$ **then** inverse-list($i - 1, p$)
 else cons(i , cons(inverse(i, p), inverse-list($i - 1, p$))) **endif**

THEOREM: all-non-zero-inverse-list

$$(\text{prime}(p) \wedge (i < (p - 1))) \rightarrow \text{all-non-zero}(\text{inverse-list}(i, p))$$

THEOREM: bounded-inverse-list

$$(\text{prime}(p) \wedge (i < (p - 1)) \wedge (j = (p - 2))) \\ \rightarrow \text{all-lesseq}(\text{inverse-list}(i, p), j)$$

THEOREM: subsetp-positives

$$\text{subsetp}(\text{positives}(n), \text{inverse-list}(n, p))$$

THEOREM: inverse-1

$$(1 < p) \rightarrow (\text{inverse}(1, p) = 1)$$

THEOREM: a-d-i-l-lemma1

$$(\text{prime}(p) \wedge ((i \bmod p) \neq 0) \wedge (i < p) \wedge (j \in \text{inverse-list}(i, p))) \\ \rightarrow (\text{inverse}(j, p) \in \text{inverse-list}(i, p))$$

THEOREM: a-d-i-l-lemma2

$$(\text{prime}(p) \\ \wedge ((i \bmod p) \neq 0) \\ \wedge ((j \bmod p) \neq 0) \\ \wedge (i < p) \\ \wedge (j < p) \\ \wedge (\text{inverse}(j, p) \in \text{inverse-list}(i, p))) \\ \rightarrow (j \in \text{inverse-list}(i, p))$$

THEOREM: a-d-i-l-lemma3

$$(\text{prime}(p) \wedge (i < (p - 1)) \wedge \text{all-distinct}(\text{inverse-list}(i - 1, p))) \\ \rightarrow \text{all-distinct}(\text{inverse-list}(i, p))$$

THEOREM: all-distinct-inverse-list

$$(\text{prime}(p) \wedge (i < (p - 1))) \rightarrow \text{all-distinct}(\text{inverse-list}(i, p))$$

THEOREM: t-i-l-lemma1

$$(((a * b) \bmod p) = 1) \rightarrow (((a * (b * c)) \bmod p) = (c \bmod p))$$

THEOREM: t-i-l-lemma

$$\begin{aligned} & (((i * \text{inverse}(i, p)) \bmod p) = 1) \\ \rightarrow & ((\text{times-list}(\text{inverse-list}(i, p)) \bmod p) \\ & = (\text{times-list}(\text{inverse-list}(i - 1, p)) \bmod p)) \end{aligned}$$

THEOREM: t-i-l-lemma3

$$\begin{aligned} & (\text{prime}(p) \wedge ((i \bmod p) \neq 0)) \\ \rightarrow & ((\text{times-list}(\text{inverse-list}(i, p)) \bmod p) \\ & = (\text{times-list}(\text{inverse-list}(i - 1, p)) \bmod p)) \end{aligned}$$

THEOREM: t-i-l-lemma4

$$(i \leq 1) \rightarrow (\text{times-list}(\text{inverse-list}(i, p)) = 1)$$

THEOREM: times-inverse-list

$$(\text{prime}(p) \wedge (i < p)) \rightarrow ((\text{times-list}(\text{inverse-list}(i, p)) \bmod p) = 1)$$

THEOREM: delete-x-leave-a

$$((a \in s) \wedge (a \neq x)) \rightarrow (a \in \text{delete}(x, s))$$

THEOREM: delete-member-leave-subset

$$(\text{subsetp}(r, s) \wedge (x \notin r)) \rightarrow \text{subsetp}(r, \text{delete}(x, s))$$

THEOREM: all-lesseqp-delete

$$(\text{all-distinct}(l) \wedge \text{all-lesseqp}(l, n)) \rightarrow \text{all-lesseqp}(\text{delete}(n, l), n - 1)$$

THEOREM: positives-bounded

$$(n < m) \rightarrow (m \notin \text{positives}(n))$$

THEOREM: subsetp-positives-delete

$$\text{subsetp}(\text{positives}(n), l) \rightarrow \text{subsetp}(\text{positives}(n - 1), \text{delete}(n, l))$$

THEOREM: nonzerop-lesseqp-zero

$$((n \simeq 0) \wedge \text{all-lesseqp}(l, n) \wedge \text{all-non-zerop}(l)) \rightarrow (\neg \text{listp}(l))$$

DEFINITION:

pigeonhole2-induction(l, n)

= **if** $n \simeq 0$ **then** **t**
 else pigeonhole2-induction($\text{delete}(n, l), n - 1$) **endif**

THEOREM: pigeonhole2

$$\begin{aligned} & (\text{all-distinct}(l) \\ & \wedge \text{all-non-zerop}(l) \\ & \wedge \text{all-lesseqp}(l, n) \\ & \wedge \text{subsetp}(\text{positives}(n), l)) \\ \rightarrow & \text{perm}(\text{positives}(n), l) \end{aligned}$$

THEOREM: perm-positives-inverse-list
 $(\text{prime}(p) \wedge (i = (p - 2))) \rightarrow \text{perm}(\text{positives}(i), \text{inverse-list}(i, p))$

THEOREM: inverse-list-fact
 $(\text{prime}(p) \wedge (i = (p - 2)))$
 $\rightarrow (\text{times-list}(\text{inverse-list}(i, p)) = \text{fact}(i))$

THEOREM: w-t-lemma
 $(\text{prime}(p) \wedge (i = (p - 2))) \rightarrow ((\text{fact}(i) \bmod p) = 1)$

THEOREM: wilson-thm
 $\text{prime}(p) \rightarrow ((\text{fact}(p - 1) \bmod p) = (p - 1))$

EVENT: Make the library "**wilson**" and compile it.

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