```
#|

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|#
```

EVENT: Start with the library "mlp" using the compiled version.

```
; funacc.bm
;   Circuit is similar to acc, but uses ARBITRARY CHAR-FUN (arity 2) instead of
; addition. It's expressed in CSXA form.  Proving it guarantees (at the BM
; level) that the theorems stated in bm_syds_2ndorder.txt are indeed true
; in general, although of course, BM can not instantiate them.
;

;;; DEFINITION OF CIRCUIT:
#|
(setq sysd '(sy-funacc (x)
(Yfun S Fun2 x Yreg)
```

```
(Yreg R 'finit Yfun)
))

(setq funacc '(
|#
; BM DEFINITIONS and A2 LEMMAS, generated by BMSYSD:
; comb_fun2.bm: Fun2 combinational element
; U7-DONE

; arbitrary Char-Fun of arity 2:
```

EVENT: Introduce the function symbol *fun2* of 2 arguments.

```
; Everything below generated by: (bmcomb 'Fun2 '() '(x y))
```

DEFINITION:
s-fun2 $(x,\ y)$
$=$ **if** empty (x) **then** E
else a (s-fun2 (p (x), p (y)), fun2 (l (x), l (y))) **endif**

```
;; A2-Begin-S-FUN2
```

THEOREM: a2-empty-s-fun2
empty (s-fun2 $(x,\ y)$) $=$ empty (x)

THEOREM: a2-e-s-fun2
(s-fun2 $(x,\ y)$ $=$ E) $=$ empty (x)

THEOREM: a2-lp-s-fun2
len (s-fun2 $(x,\ y)$) $=$ len (x)

THEOREM: a2-lpe-s-fun2
eqlen (s-fun2 $(x,\ y)$, x)

THEOREM: a2-ic-s-fun2
(len (x) $=$ len (y))
$\rightarrow$ (s-fun2 (i $(c_x,\ x)$, i $(c_y,\ y)$) $=$ i (fun2 $(c_x,\ c_y)$, s-fun2 $(x,\ y)$))

THEOREM: a2-lc-s-fun2
($\neg$ empty (x)) $\rightarrow$ (l (s-fun2 $(x,\ y)$) $=$ fun2 (l (x), l (y)))

THEOREM: a2-pc-s-fun2
p (s-fun2 $(x,\ y)$) $=$ s-fun2 (p (x), p (y))

THEOREM: a2-hc-s-fun2
((¬ empty (x)) ∧ (len (x) = len (y)))
→ (h (s-fun2 (x, y)) = fun2 (h (x), h (y)))

THEOREM: a2-bc-s-fun2
(len (x) = len (y)) → (b (s-fun2 (x, y)) = s-fun2 (b (x), b (y)))

THEOREM: a2-bnc-s-fun2
(len (x) = len (y)) → (bn (n, s-fun2 (x, y)) = s-fun2 (bn (n, x), bn (n, y)))

```
;; A2-End-S-FUN2

; eof:comb_fun2.bm
```

DEFINITION:
topor-sy-funacc (ln)
= **if** ln = `'yfun` **then** `1`
elseif ln = `'yreg` **then** `0`
else `0` **endif**

DEFINITION:
sy-funacc (ln, x)
= **if** ln = `'yfun` **then** s-fun2 (x, sy-funacc (`'yreg`, x))
elseif ln = `'yreg`
then if empty (x) **then** E
else i (`'finit`, sy-funacc (`'yfun`, p (x))) **endif**
else sfix (x) **endif**

```
;; A2-Begin-SY-FUNACC
```

THEOREM: a2-empty-sy-funacc
empty (sy-funacc (ln, x)) = empty (x)

THEOREM: a2-e-sy-funacc
(sy-funacc (ln, x) = E) = empty (x)

THEOREM: a2-lp-sy-funacc
len (sy-funacc (ln, x)) = len (x)

THEOREM: a2-lpe-sy-funacc
eqlen (sy-funacc (ln, x), x)

THEOREM: a2-pc-sy-funacc
p (sy-funacc (ln, x)) = sy-funacc (ln, p (x))

```
;; A2-End-SY-FUNACC

;;; SPEC definition:
```

DEFINITION:
numer-funacc (x)
$=$ **if** empty (x) **then** `'finit`
else fun2 (numer-funacc (p (x)), l (x)) **endif**

```
; this is the standard extension from last-char-fun to MLP-string-fun.
```

DEFINITION:
spec-funacc (x)
$=$ **if** empty (x) **then** E
else a (spec-funacc (p (x)), numer-funacc (x)) **endif**

```
;;; Circuit CORRECTNESS:

; We now declare Fun2 to be COMMUTATIVE because the definition of NUMER-FUNACC
; funs the 2 arguments in the opposite order than the sysd, and hence
; commutativity is required.  Alternatively, we could change NUMER-FUNACC's
; definition.  (This has been tested and runs.)
```

AXIOM: fun2-comm
fun2 (u, v) = fun2 (v, u)

```
; Funacc-correct-ax is a "predicative correctness statement",i.e. what we would
; do if we didn't have functional equality as a specification method, but
; instead used a purely axiomatic approach.
; NOTE: the simplicity of the proof should imply that disabling the
; char-function in specific instances of accumulator syds should help...
```

THEOREM: funacc-correct-ax
$(\neg$ empty $(x)) \rightarrow$ (l (sy-funacc (`'yfun`, x)) = numer-funacc (x))

```
; to go to a functional equality once we have the "last" (ax) statement is
; a trivial induction, if we start out with an P-L split which is unnatural
; for BM, so we force it w/ a USE hint of A-p-l-split .  See also THETA.BM .
```

THEOREM: a-p-l-split
$(\neg$ empty $(x))$
$\rightarrow$ (sy-funacc (`'yfun`, x)
$=$ a (p (sy-funacc (`'yfun`, x)), l (sy-funacc (`'yfun`, x))))

THEOREM: funacc-correct
sy-funacc ('`yfun`, x) = spec-funacc (x)

```
; eof: funacc.bm
;))
```

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