```
#|

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|#
```

EVENT: Start with the library "mlp" using the compiled version.

```
; pplfun3.bm is our 2nd PIPELINE proof: it generalizes Inc to FUN1.
;
; Note: we run into trouble w/ R-loop corking, because of the DCL'd function.
; A partial kludge is to:
; (LOAD "Comb/comb_fun1_s.bm") , i.e. with the _s extension, getting an actual
; symbolic value (a list).  But we really need full BY NAME evaluation..

;;; (Sugared) Circuits:
#|
(setq A '(SY-A (x)
(Y1 S Fun1 x)
```

```
(Y2 S Fun1 Y1)
(Y3 S Fun1 Y2)
; and the cork:
(Yc2 R (FUN1 (FUN1 0)) Y3)
(Yc1 R (FUN1 0) Yc2)
(Yout R 0 Yc1)
))

(setq B '(SY-B (x)
(Z1 S Fun1 x)
(Z2 R 0 Z1)
(Z3 S Fun1 Z2)
(Z4 R 0 Z3)
(Z5 S Fun1 Z4)
(Zout R 0 Z5)
))

(setq pplfun3 '( |#
; BM DEFINITIONS and A2 LEMMAS, generated by BMSYSD:
; comb_fun1.bm: Fun1 combinational element
; U7-DONE

; arbitrary Char-Fun of arity 1:
```

EVENT: Introduce the function symbol *fun1* of one argument.

```
; Everything below generated by: (bmcomb 'Fun1 '() '(x))
```

DEFINITION:
s-fun1 (x)
$=$ **if** empty (x) **then** E
else a (s-fun1 (p (x)), fun1 (l (x))) **endif**

```
;; A2-Begin-S-FUN1
```

THEOREM: a2-empty-s-fun1
empty (s-fun1 (x)) $=$ empty (x)

THEOREM: a2-e-s-fun1
(s-fun1 (x) $=$ E) $=$ empty (x)

THEOREM: a2-lp-s-fun1
len (s-fun1 (x)) $=$ len (x)

THEOREM: a2-lpe-s-fun1
eqlen (s-fun1 (x), x)

THEOREM: a2-ic-s-fun1
s-fun1 (i (c_x, x)) = i (fun1 (c_x), s-fun1 (x))

THEOREM: a2-lc-s-fun1
($\neg$ empty (x)) $\rightarrow$ (l (s-fun1 (x)) = fun1 (l (x)))

THEOREM: a2-pc-s-fun1
p (s-fun1 (x)) = s-fun1 (p (x))

THEOREM: a2-hc-s-fun1
($\neg$ empty (x)) $\rightarrow$ (h (s-fun1 (x)) = fun1 (h (x)))

THEOREM: a2-bc-s-fun1
b (s-fun1 (x)) = s-fun1 (b (x))

THEOREM: a2-bnc-s-fun1
bn (n, s-fun1 (x)) = s-fun1 (bn (n, x))

```
;; A2-End-S-FUN1

; eof:comb_fun1.bm
```

DEFINITION:
topor-sy-a (ln)
= **if** ln = 'y1 **then** 1
elseif ln = 'y2 **then** 2
elseif ln = 'y3 **then** 3
elseif ln = 'yc2 **then** 0
elseif ln = 'yc1 **then** 0
elseif ln = 'yout **then** 0
else 0 **endif**

DEFINITION:
sy-a (ln, x)
= **if** ln = 'y1 **then** s-fun1 (x)
elseif ln = 'y2 **then** s-fun1 (sy-a ('y1, x))
elseif ln = 'y3 **then** s-fun1 (sy-a ('y2, x))
elseif ln = 'yc2
then if empty (x) **then** E
else i (fun1 (fun1 (0)), sy-a ('y3, p (x))) **endif**
elseif ln = 'yc1

```
    then if empty (x)  then E
          else i (fun1 (0), sy-a ('yc2, p (x))) endif
    elseif ln = 'yout
    then if empty (x)  then E
          else i (0, sy-a ('yc1, p (x))) endif
    else sfix (x) endif

;; A2-Begin-SY-A
```

THEOREM: a2-empty-sy-a
empty (sy-a (*ln*, *x*)) = empty (*x*)

THEOREM: a2-e-sy-a
(sy-a (*ln*, *x*) = E) = empty (*x*)

THEOREM: a2-lp-sy-a
len (sy-a (*ln*, *x*)) = len (*x*)

THEOREM: a2-lpe-sy-a
eqlen (sy-a (*ln*, *x*), *x*)

THEOREM: a2-pc-sy-a
p (sy-a (*ln*, *x*)) = sy-a (*ln*, p (*x*))

```
;; A2-End-SY-A
```

DEFINITION:
topor-sy-b (*ln*)

```
=   if ln = 'z1 then 1
    elseif ln = 'z2 then 0
    elseif ln = 'z3 then 1
    elseif ln = 'z4 then 0
    elseif ln = 'z5 then 1
    elseif ln = 'zout then 0
    else 0 endif
```

DEFINITION:
sy-b (*ln*, *x*)

```
=   if ln = 'z1 then s-fun1 (x)
    elseif ln = 'z2
    then if empty (x)  then E
          else i (0, sy-b ('z1, p (x))) endif
    elseif ln = 'z3 then s-fun1 (sy-b ('z2, x))
    elseif ln = 'z4
```

then if empty (x) **then** E
  else i (0, sy-b ('z3, p (x))) **endif**
elseif ln = 'z5 **then** s-fun1 (sy-b ('z4, x))
elseif ln = 'zout
then if empty (x) **then** E
  else i (0, sy-b ('z5, p (x))) **endif**
else sfix (x) **endif**

```
;; A2-Begin-SY-B
```

THEOREM: a2-empty-sy-b
empty (sy-b (ln, x)) = empty (x)

THEOREM: a2-e-sy-b
(sy-b (ln, x) = E) = empty (x)

THEOREM: a2-lp-sy-b
len (sy-b (ln, x)) = len (x)

THEOREM: a2-lpe-sy-b
eqlen (sy-b (ln, x), x)

THEOREM: a2-pc-sy-b
p (sy-b (ln, x)) = sy-b (ln, p (x))

```
;; A2-End-SY-B

;;; CORRECTNESS PROOF (hand generated, dreamer!):


; EQ-A-B:
;   We start out directly with the expansion list form pplinc3.
```

THEOREM: eq-a-b
sy-b ('zout, x) = sy-a ('yout, x)

```
; eof: pplfun3.bm
;))
```

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