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|#

EVENT: Start with the library "mlp" using the compiled version.

```
; prod0_CSXAdd.bm
; . definition of circuits [assumes stringadd.bm] :
        - if the circuit has only one line: OK without any hint
; We MAY want to put the TOPO hint, just for the induction, although
; for one line it probably collapes to the same induction (LEN X).
; - if the circuit has more than one line:
; - without hints: FAIL
; - TOPOO is not definable, because of loops in the dependency graph!
; - with TOPOR: OK
; NOTE: the above comments date back to the hand-generation time, when we
; were still trying to FIND a way to feed things to BM. They are kept
```

#|

```
here for historical purposes only...
;
;;; DEFINITION OF CIRCUITS:
#|
(setq sysd-prod '(sy-prod (x)
(Yprod S Times x Yprod2)
(Yprod2 R 0 Yprod)
))
(setq sysd-const0 '(sy-const0 (x)
(YconstO R O YconstO)
))
(setq prod0_CSXA00 '(
|#
; BM DEFINITIONS and A2 LEMMAS, generated by BMSYSD:
; comb_times.bm: Times combinational element.
; U7-DONE
; no character function def since BM already knows about Times..
; Everything below generated by:
                                        (bmcomb 'times '() '(x y))
DEFINITION:
s-times (x, y)
= if empty (x) then E
   else a (s-times (p(x), p(y)), l(x) * l(y)) endif
;; A2-Begin-S-TIMES
THEOREM: a2-empty-s-times
empty(s-times(x, y)) = empty(x)
THEOREM: a2-e-s-times
(s-times(x, y) = E) = empty(x)
THEOREM: a2-lp-s-times
\ln\left(\text{s-times}\left(x, y\right)\right) = \ln\left(x\right)
THEOREM: a2-lpe-s-times
eqlen (s-times (x, y), x)
```

THEOREM: a2-ic-s-times $(\operatorname{len}(x) = \operatorname{len}(y))$ \rightarrow (s-times (i (c_x, x), i (c_y, y)) = i ($c_x * c_y$, s-times (x, y))) THEOREM: a2-lc-s-times $(\neg \operatorname{empty}(x)) \rightarrow (\operatorname{l}(\operatorname{s-times}(x, y)) = (\operatorname{l}(x) * \operatorname{l}(y)))$ THEOREM: a2-pc-s-times p(s-times(x, y)) = s-times(p(x), p(y))**THEOREM:** a2-hc-s-times $\left(\left(\neg \operatorname{empty}\left(x\right)\right) \land \left(\operatorname{len}\left(x\right) = \operatorname{len}\left(y\right)\right)\right)$ $\rightarrow \quad (\mathbf{h}\left(\mathbf{s}\text{-times}\left(x,\,y\right)\right) = \left(\mathbf{h}\left(x\right)\,\ast\,\mathbf{h}\left(y\right)\right))$ THEOREM: a2-bc-s-times $(\operatorname{len}(x) = \operatorname{len}(y)) \to (\operatorname{b}(\operatorname{s-times}(x, y)) = \operatorname{s-times}(\operatorname{b}(x), \operatorname{b}(y)))$ THEOREM: a2-bnc-s-times $(\operatorname{len}(x) = \operatorname{len}(y)) \to (\operatorname{bn}(n, \operatorname{s-times}(x, y)) = \operatorname{s-times}(\operatorname{bn}(n, x), \operatorname{bn}(n, y)))$;; A2-End-S-TIMES ; eof:comb_times.bm **DEFINITION:** topor-sy-prod (ln)= if ln = 'yprod then 1 elseif ln = 'yprod2 then 0 else 0 endif **DEFINITION:** sy-prod (ln, x)= if ln ='yprod then s-times (x, sy-prod('yprod2, x))elseif ln = 'yprod2then if empty(x) then E else i (0, sy-prod('yprod, p(x))) endif else sfix (x) endif ;; A2-Begin-SY-PROD THEOREM: a2-empty-sy-prod empty(sy-prod(ln, x)) = empty(x)

```
THEOREM: a2-e-sy-prod
(\text{sy-prod}(ln, x) = E) = \text{empty}(x)
THEOREM: a2-lp-sy-prod
\ln\left(\text{sy-prod}\left(ln, x\right)\right) = \ln\left(x\right)
THEOREM: a2-lpe-sy-prod
eqlen (sy-prod (ln, x), x)
THEOREM: a2-pc-sy-prod
p(sy-prod(ln, x)) = sy-prod(ln, p(x))
;; A2-End-SY-PROD
; BM DEFINITIONS and A2 LEMMAS, generated by BMSYSD:
; No TOPO def for 1 line sysds because it is not needed and confuses BM
DEFINITION:
sy-const0(ln, x)
= if ln = 'yconst0
    then if empty(x) then E
          else i (0, sy-const0('yconst0, p(x))) endif
    else sfix (x) endif
;; A2-Begin-SY-CONSTO
THEOREM: a2-empty-sy-const0
empty(sy-const0(ln, x)) = empty(x)
THEOREM: a2-e-sy-const0
(\text{sy-const0}(ln, x) = E) = \text{empty}(x)
THEOREM: a2-lp-sy-const0
\ln\left(\text{sy-const0}\left(ln,\,x\right)\right) = \ln\left(x\right)
THEOREM: a2-lpe-sy-const0
eqlen (sy-const0 (ln, x), x)
THEOREM: a2-pc-sy-const0
p(sy-const0(ln, x)) = sy-const0(ln, p(x))
;; A2-End-SY-CONSTO
;;; PROOF OF EQUIVALENCE:
; The key fact about SY-Yconst is that it equals the constant 0 function:
```

THEOREM: sy-const0-is-const sy-const0('yconst0, x) = s-const(0, x)

; The key fact (bug) about prod0 is that both lines also equal const-0 sfun ; CRUCIAL NOTE: we only want the 1st equality, but in order for the induction ; proof to succeed, we need the stronger (global) statement.

```
THEOREM: prod0-is-const
(sy-prod('yprod, x) = s-const(0, x))
\land (sy-prod('yprod2, x) = s-const(0, x))
```

```
; now the equality is trivial:
```

```
THEOREM: e_prodconst0
sy-prod('yprod, x) = sy-const0('yconst0, x)
```

; eof: prod0_CSXA00.bm
;))

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