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EVENT: Start with the initial **nqthm** theory.

DEFINITION: $\text{logicalp}(x) = ((x = \text{TRUE}) \vee (x = \text{FALSE}))$

DEFINITION:

$\text{expt}(i, j)$
= **if** $j \simeq 0$ **then** 1
 else $i * \text{expt}(i, j - 1)$ **endif**

DEFINITION: $\text{znumberp}(x) = (\text{negativep}(x) \vee (x \in \mathbf{N}))$

DEFINITION: $\text{ZZERO} = \text{ZERO}$

DEFINITION:

$\text{zplus}(x, y)$
= **if** $\text{negativep}(x)$

```

then if negativep( $y$ ) then  $-(\text{negative-guts}(x) + \text{negative-guts}(y))$ 
    elseif  $y < \text{negative-guts}(x)$  then  $-(\text{negative-guts}(x) - y)$ 
    else  $y - \text{negative-guts}(x)$  endif
elseif negativep( $y$ )
then if  $x < \text{negative-guts}(y)$  then  $-(\text{negative-guts}(y) - x)$ 
    else  $x - \text{negative-guts}(y)$  endif
else  $x + y$  endif

```

DEFINITION:

zdifference(x, y)

```

= if negativep( $x$ )
  then if negativep( $y$ )
    then if  $\text{negative-guts}(y) < \text{negative-guts}(x)$ 
      then  $-(\text{negative-guts}(x) - \text{negative-guts}(y))$ 
      else  $\text{negative-guts}(y) - \text{negative-guts}(x)$  endif
    else  $-(\text{negative-guts}(x) + y)$  endif
  elseif negativep( $y$ ) then  $x + \text{negative-guts}(y)$ 
  elseif  $x < y$  then  $-(y - x)$ 
  else  $x - y$  endif

```

DEFINITION:

ztimes(x, y)

```

= if negativep( $x$ )
  then if negativep( $y$ ) then  $\text{negative-guts}(x) * \text{negative-guts}(y)$ 
    else  $-(\text{negative-guts}(x) * y)$  endif
  elseif negativep( $y$ ) then  $-(x * \text{negative-guts}(y))$ 
  else  $x * y$  endif

```

DEFINITION:

zquotient(x, y)

```

= if negativep( $x$ )
  then if negativep( $y$ ) then  $\text{negative-guts}(x) \div \text{negative-guts}(y)$ 
    else  $-(\text{negative-guts}(x) \div y)$  endif
  elseif negativep( $y$ ) then  $-(x \div \text{negative-guts}(y))$ 
  else  $x \div y$  endif

```

DEFINITION:

zexptz(i, j)

```

= if  $j \simeq 0$  then 1
  else ztimes( $i, \text{zexptz}(i, j - 1)$ ) endif

```

DEFINITION:

znormalize(x)

```

= if negativep( $x$ )
  then if  $\text{negative-guts}(x) = 0$  then 0

```

else x endif
else fix(x) endif

DEFINITION: $zeqp(x, y) = (znormalize(x) = znormalize(y))$

DEFINITION: $zneqp(x, y) = (\neg zeqp(x, y))$

DEFINITION:

$zlessp(x, y)$

= **if** negativep(x)

then if negativep(y) **then** negative-guts(y) < negative-guts(x)

else $\neg ((\text{negative-guts}(x) = 0) \wedge (y \simeq 0))$ **endif**

elseif negativep(y) **then f**

else $x < y$ endif

DEFINITION: $zlesseqp(x, y) = (\neg zlessp(y, x))$

DEFINITION: $zgreaterp(x, y) = zlessp(y, x)$

DEFINITION: $zgreaterreqp(x, y) = (\neg zlessp(x, y))$

CONSERVATIVE AXIOM: integer-size

(LEAST-INEXPRESSIBLE-POSITIVE-INTEGER $\in \mathbf{N}$)

\wedge negativep(GREATEST-INEXPRESSIBLE-NEGATIVE-INTEGER)

\wedge (200 < negative-guts(GREATEST-INEXPRESSIBLE-NEGATIVE-INTEGER))

\wedge (200 < LEAST-INEXPRESSIBLE-POSITIVE-INTEGER)

Simultaneously, we introduce the new function symbols *greatest-inexpressible-negative-integer* and *least-inexpressible-positive-integer*.

DEFINITION:

expressible-znumberp(x)

= (zlessp(GREATEST-INEXPRESSIBLE-NEGATIVE-INTEGER, x)

\wedge zlessp(x , LEAST-INEXPRESSIBLE-POSITIVE-INTEGER))

DEFINITION:

iabs(i)

= **if** negativep(i) **then** negative-guts(i)

else fix(i) endif

DEFINITION: $\text{mod}(x, y) = \text{zdifference}(x, \text{ztimes}(y, \text{zquotient}(x, y)))$

DEFINITION:

$\text{max0}(i, j)$

= **if** zlessp(i, j) **then** j

else i endif

DEFINITION:

```
min0(i, j)
=  if zlessp(i, j) then i
    else j endif
```

DEFINITION:

```
isign(i, j)
=  if negativep(j) then ztimes(-1, iabs(i))
    else iabs(i) endif
```

DEFINITION: $\text{idim}(i, j) = \text{zdifference}(i, \text{min0}(i, j))$

```
;;; In the old FORTRAN xxx we used UNDEF for all occurrences of the
;;; substring FORTRAN-UNDEF below. However, in the quantifier version
;;; of the logic, UNDEF is a function name set up in the bootstrap.
;;; To avoid conflict we have changed the name here. However, it is
;;; possible that vcs mention UNDEF -- not just the function DEFINEDP
;;; which is still around here. If so, the vcg will have to be
;;; changed should we decide to support it.
```

EVENT: Add the shell *fortran-undef*, with recognizer function symbol *fortran-undefined* and 1 accessor: *fortran-undef-guts*, with type restriction (none-of) and default value zero.

DEFINITION: $\text{definedp}(x) = (\neg \text{fortran-undefined}(x))$

EVENT: Introduce the function symbol *elt1* of 2 arguments.

EVENT: Introduce the function symbol *elt2* of 3 arguments.

EVENT: Introduce the function symbol *elt3* of 4 arguments.

DEFINITION:

```
lex(l1, l2)
=  if (l1  $\simeq$  nil)  $\vee$  (l2  $\simeq$  nil) then f
    else (car(l1) < car(l2))
         $\vee$  ((car(l1) = car(l2))  $\wedge$  lex(cdr(l1), cdr(l2))) endif
```

EVENT: Introduce the function symbol *rnumberp* of one argument.

EVENT: Introduce the function symbol *dnumberp* of one argument.

EVENT: Introduce the function symbol *cnumberp* of one argument.

EVENT: Introduce the function symbol *rzero* of 0 arguments.

EVENT: Introduce the function symbol *dzero* of 0 arguments.

EVENT: Introduce the function symbol *czero* of 0 arguments.

EVENT: Introduce the function symbol *expressible-rnumberp* of one argument.

EVENT: Introduce the function symbol *expressible-dnumberp* of one argument.

EVENT: Introduce the function symbol *expressible-cnumberp* of one argument.

EVENT: Introduce the function symbol *rplus* of 2 arguments.

EVENT: Introduce the function symbol *rtimes* of 2 arguments.

EVENT: Introduce the function symbol *rdifference* of 2 arguments.

EVENT: Introduce the function symbol *rquotient* of 2 arguments.

EVENT: Introduce the function symbol *rlessp* of 2 arguments.

EVENT: Introduce the function symbol *rlesseqp* of 2 arguments.

EVENT: Introduce the function symbol *reqp* of 2 arguments.

EVENT: Introduce the function symbol *rneqp* of 2 arguments.

EVENT: Introduce the function symbol *rgreatereqp* of 2 arguments.

EVENT: Introduce the function symbol *rgreaterp* of 2 arguments.

EVENT: Introduce the function symbol *dplus* of 2 arguments.

EVENT: Introduce the function symbol *dtimes* of 2 arguments.

EVENT: Introduce the function symbol *ddifference* of 2 arguments.

EVENT: Introduce the function symbol *dquotient* of 2 arguments.

EVENT: Introduce the function symbol *dlessp* of 2 arguments.

EVENT: Introduce the function symbol *dlesseqp* of 2 arguments.

EVENT: Introduce the function symbol *deqp* of 2 arguments.

EVENT: Introduce the function symbol *dneqp* of 2 arguments.

EVENT: Introduce the function symbol *dgreaterqp* of 2 arguments.

EVENT: Introduce the function symbol *dgreaterp* of 2 arguments.

EVENT: Introduce the function symbol *cplus* of 2 arguments.

EVENT: Introduce the function symbol *ctimes* of 2 arguments.

EVENT: Introduce the function symbol *cdifference* of 2 arguments.

EVENT: Introduce the function symbol *cquotient* of 2 arguments.

EVENT: Introduce the function symbol *ceqp* of 2 arguments.

EVENT: Introduce the function symbol *cneqp* of 2 arguments.

EVENT: Introduce the function symbol *rexptz* of 2 arguments.

EVENT: Introduce the function symbol *dexptz* of 2 arguments.

EVENT: Introduce the function symbol *cexptz* of 2 arguments.

EVENT: Introduce the function symbol *rexp* of 2 arguments.

EVENT: Introduce the function symbol *rexp**td* of 2 arguments.

EVENT: Introduce the function symbol *dexp* of 2 arguments.

EVENT: Introduce the function symbol *dexp**td* of 2 arguments.

EVENT: Introduce the function symbol *abs* of one argument.

EVENT: Introduce the function symbol *dabs* of one argument.

EVENT: Introduce the function symbol *aint* of one argument.

EVENT: Introduce the function symbol *int* of one argument.

EVENT: Introduce the function symbol *idint* of one argument.

EVENT: Introduce the function symbol *amod* of 2 arguments.

EVENT: Introduce the function symbol *amax0* of 2 arguments.

EVENT: Introduce the function symbol *amax1* of 2 arguments.

EVENT: Introduce the function symbol *max1* of 2 arguments.

EVENT: Introduce the function symbol *dmax1* of 2 arguments.

EVENT: Introduce the function symbol *amin0* of 2 arguments.

EVENT: Introduce the function symbol *amin1* of 2 arguments.

EVENT: Introduce the function symbol *min1* of 2 arguments.

EVENT: Introduce the function symbol *dmin1* of 2 arguments.

EVENT: Introduce the function symbol *float* of one argument.

EVENT: Introduce the function symbol *ifix* of one argument.

EVENT: Introduce the function symbol *sign* of 2 arguments.

EVENT: Introduce the function symbol *dsign* of 2 arguments.

EVENT: Introduce the function symbol *dim* of 2 arguments.

EVENT: Introduce the function symbol *sngl* of one argument.

EVENT: Introduce the function symbol *real* of one argument.

EVENT: Introduce the function symbol *aimag* of one argument.

EVENT: Introduce the function symbol *dbl* of one argument.

EVENT: Introduce the function symbol *cmplx* of 2 arguments.

EVENT: Introduce the function symbol *conjg* of one argument.

EVENT: Introduce the function symbol *exp* of one argument.

EVENT: Introduce the function symbol *dexp* of one argument.

EVENT: Introduce the function symbol *cexp* of one argument.

EVENT: Introduce the function symbol *alog* of one argument.

EVENT: Introduce the function symbol *dlog* of one argument.

EVENT: Introduce the function symbol *clog* of one argument.

EVENT: Introduce the function symbol *alog10* of one argument.

EVENT: Introduce the function symbol $dlog10$ of one argument.

EVENT: Introduce the function symbol sin of one argument.

EVENT: Introduce the function symbol $dsin$ of one argument.

EVENT: Introduce the function symbol $csin$ of one argument.

EVENT: Introduce the function symbol cos of one argument.

EVENT: Introduce the function symbol $dcos$ of one argument.

EVENT: Introduce the function symbol $ccos$ of one argument.

EVENT: Introduce the function symbol $tanh$ of one argument.

EVENT: Introduce the function symbol $sqrt$ of one argument.

EVENT: Introduce the function symbol $dsqrt$ of one argument.

EVENT: Introduce the function symbol $csqrt$ of one argument.

EVENT: Introduce the function symbol $atan$ of one argument.

EVENT: Introduce the function symbol $datan$ of one argument.

EVENT: Introduce the function symbol $atan2$ of 2 arguments.

EVENT: Introduce the function symbol $datan2$ of 2 arguments.

EVENT: Introduce the function symbol $dmod$ of 2 arguments.

EVENT: Introduce the function symbol $cabs$ of one argument.

DEFINITION:

$$\begin{aligned}
& \text{almost-equal1}(a1, a2, u, v, i, e) \\
= & \text{if } (v \simeq 0) \vee (v < u) \text{ then } \mathbf{t} \\
& \quad \text{else if } v = i \text{ then elt1}(a2, v) = e \\
& \quad \quad \text{else elt1}(a2, v) = \text{elt1}(a1, v) \text{ endif} \\
& \quad \quad \wedge \text{almost-equal1}(a1, a2, u, v - 1, i, e) \text{ endif}
\end{aligned}$$

THEOREM: plus-0
 $(x + 0) = \text{fix}(x)$

THEOREM: plus-non-numberp
 $(y \notin \mathbf{N}) \rightarrow ((x + y) = \text{fix}(x))$

THEOREM: plus-add1
 $(x + (1 + y))$
 $= \text{if } y \in \mathbf{N} \text{ then } 1 + (x + y)$
 $\quad \text{else } 1 + x \text{ endif}$

THEOREM: commutativity2-of-plus
 $(x + (y + z)) = (y + (x + z))$

THEOREM: commutativity-of-plus
 $(x + y) = (y + x)$

THEOREM: associativity-of-plus
 $((x + y) + z) = (x + (y + z))$

THEOREM: times-0
 $(x * 0) = 0$

THEOREM: times-non-numberp
 $(y \notin \mathbf{N}) \rightarrow ((x * y) = 0)$

THEOREM: distributivity-of-times-over-plus
 $(x * (y + z)) = ((x * y) + (x * z))$

THEOREM: times-add1
 $(x * (1 + y))$
 $= \text{if } y \in \mathbf{N} \text{ then } x + (x * y)$
 $\quad \text{else } \text{fix}(x) \text{ endif}$

THEOREM: commutativity2-of-times
 $(x * (y * z)) = (y * (x * z))$

THEOREM: commutativity-of-times
 $(x * y) = (y * x)$

THEOREM: associativity-of-times

$$((x * y) * z) = (x * (y * z))$$

THEOREM: equal-times-0

$$((x * y) = 0) = ((x \simeq 0) \vee (y \simeq 0))$$

THEOREM: equal-lessp

$$\begin{aligned} & ((x < y) = z) \\ = & \text{ if } x < y \text{ then } t = z \\ & \text{ else } f = z \text{ endif} \end{aligned}$$

THEOREM: almost-equal1-in-range

$$\begin{aligned} & ((\text{elt1}(a2, j) \neq w) \\ & \wedge (w = \text{if } j = i \text{ then } e \\ & \quad \text{else elt1}(a1, j) \text{ endif}) \\ & \wedge (u \neq 0) \\ & \wedge (j \not\leq u) \\ & \wedge (v \not\leq j)) \\ \rightarrow & (\neg \text{almost-equal1}(a1, a2, u, v, i, e)) \end{aligned}$$

THEOREM: almost-equal1-in-range-opened-up

$$\begin{aligned} & ((\text{elt1}(a2, j) \neq w) \\ & \wedge (w = \text{if } j = i \text{ then } e \\ & \quad \text{else elt1}(a1, j) \text{ endif}) \\ & \wedge (u \neq 0) \\ & \wedge (u \leq j) \\ & \wedge (j \leq v) \\ & \wedge (v \neq 0) \\ & \wedge (v \not\leq u) \\ & \wedge (v \neq i) \\ & \wedge (\text{elt1}(a2, v) = \text{elt1}(a1, v))) \\ \rightarrow & (\neg \text{almost-equal1}(a1, a2, u, v - 1, i, e)) \end{aligned}$$

THEOREM: almost-equal1-contracts

$$\begin{aligned} & (\text{almost-equal1}(a1, a2, u, v, i, e) \wedge (u \neq 0) \wedge (x \not\leq u) \wedge (v \not\leq y)) \\ \rightarrow & \text{almost-equal1}(a1, a2, x, y, i, e) \end{aligned}$$

EVENT: Make the library "fortran" and compile it.

Index

abs, 7
aimag, 8
aint, 7
almost-equal1, 9–11
almost-equal1-contracts, 11
almost-equal1-in-range, 11
almost-equal1-in-range-opened-up, 11
alog, 8
alog10, 8
amax0, 7
amax1, 7
amin0, 7
amin1, 7
amod, 7
associativity-of-plus, 10
associativity-of-times, 11
atan, 9
atan2, 9

cabs, 9
ccos, 9
cdifference, 6
ceqp, 6
cexp, 8
cexptz, 6
clog, 8
cmplx, 8
cneqp, 6
cnumberp, 5
commutativity-of-plus, 10
commutativity-of-times, 10
commutativity2-of-plus, 10
commutativity2-of-times, 10
conjg, 8
cos, 9
cplus, 6
cquotient, 6
csin, 9
csqrt, 9
ctimes, 6
czero, 5

dabs, 7
datan, 9
datan2, 9
dble, 8
dcos, 9
ddifference, 6
definedp, 4
deqp, 6
dexp, 8
dexptd, 7
dexptr, 7
dexp tz, 6
dgreaterreqp, 6
dgreaterp, 6
dim, 8
distributivity-of-times-over-pl
 us, 10
dlesseqp, 6
dlessp, 6
dlog, 8
dlog10, 9
dmax1, 7
dmin1, 7
dmod, 9
dneqp, 6
dnumberp, 4
dplus, 5
dquotient, 6
dsign, 8
dsin, 9
dsqrt, 9
dtimes, 6
dzero, 5

elt1, 4, 10, 11
elt2, 4
elt3, 4
equal-lessp, 11
equal-times-0, 11
exp, 8
expressible-cnumberp, 5

expressible-dnumberp, 5
 expressible-rnumberp, 5
 expressible-znumberp, 3
 expt, 1

 float, 8
 fortran-undef, 4
 fortran-undefined, 4

 greatest-inexpressible-negative
 -integer, 3

 iabs, 3, 4
 idim, 4
 idint, 7
 ifix, 8
 int, 7
 integer-size, 3
 isign, 4

 least-inexpressible-positive-integer, 3
 lex, 4
 logicalp, 1

 max0, 3
 max1, 7
 min0, 4
 min1, 7
 mod, 3

 plus-0, 10
 plus-add1, 10
 plus-non-numberp, 10

 rdifference, 5
 real, 8
 reqp, 5
 rexptd, 7
 rexptr, 7
 rexptz, 6
 rgreatereqp, 5
 rgreaterp, 5
 rlesseqp, 5
 rlessp, 5

 rneqp, 5
 rnumberp, 4
 rplus, 5
 rquotient, 5
 rtimes, 5
 rzero, 5

 sign, 8
 sin, 9
 sngl, 8
 sqrt, 9

 tanh, 9
 times-0, 10
 times-add1, 10
 times-non-numberp, 10

 zdifference, 2–4
 zeqp, 3
 zexptz, 2
 zgreatereqp, 3
 zgreaterp, 3
 zlesseqp, 3
 zlessp, 3, 4
 zneqp, 3
 znormalize, 2, 3
 znumberp, 1
 zplus, 1
 zquotient, 2, 3
 ztimes, 2–4
 zzero, 1