EVENT: Start with the initial **nqthm** theory.

DEFINITION:
ramsey (p, q)
= **if** $p \simeq 0$ **then** 1
elseif $q \simeq 0$ **then** 1
else ramsey $(p-1, q)$ + ramsey $(p, q-1)$ **endif**

DEFINITION:
related $(i, j, pairs) = ((\text{cons}(i, j) \in pairs) \lor (\text{cons}(j, i) \in pairs))$

DEFINITION:
partition $(n, rest, pairs)$
= **if** listp $(rest)$
then if related $(n, \text{car}(rest), pairs)$
then cons (cons (car $(rest)$, car (partition $(n, \text{cdr}(rest), pairs)$)),
cdr (partition $(n, \text{cdr}(rest), pairs)$))
else cons (car (partition $(n, \text{cdr}(rest), pairs)$),
cons (car $(rest)$, cdr (partition $(n, \text{cdr}(rest), pairs)$))) **endif**
else cons (**nil**, **nil**) **endif**

DEFINITION:
length (*lst*)
= **if** listp (*lst*) **then** 1 + length (cdr (*lst*))
else 0 **endif**

DEFINITION:
wit (*pairs*, *domain*, *p*, *q*)
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=   if listp (domain)
    then if p ≃ 0  then cons (nil, 1)
           elseif q ≃ 0  then cons (nil, 2)
           elseif length (car (partition (car (domain), cdr (domain), pairs)))
                    <   ramsey (p − 1, q)
           then if cdr (wit (pairs,
                                  cdr (partition (car (domain), cdr (domain), pairs)),
                                  p,
                                  q − 1))
                     =   1
                  then wit (pairs,
                                 cdr (partition (car (domain), cdr (domain), pairs)),
                                 p,
                                 q − 1)
                  else cons (cons (car (domain),
                                          car (wit (pairs,
                                                       cdr (partition (car (domain),
                                                                              cdr (domain),
                                                                              pairs)),
                                                       p,
                                                       q − 1))),
                                 2) endif
           elseif cdr (wit (pairs,
                               car (partition (car (domain), cdr (domain), pairs)),
                               p − 1,
                               q))
                  =   2
           then wit (pairs,
                          car (partition (car (domain), cdr (domain), pairs)),
                          p − 1,
                          q)
           else cons (cons (car (domain),
                                   car (wit (pairs,
                                                car (partition (car (domain),
                                                                       cdr (domain),
                                                                       pairs)),
                                                p − 1,
```

$q))),$

1) **endif**

else cons (**nil**, 1) **endif**

DEFINITION:
homogeneous1 (n, $domain$, $pairs$, flg)
= **if** listp ($domain$)
then if $flg = 1$ **then** related (n, car ($domain$), $pairs$)
else ¬ related (n, car ($domain$), $pairs$) **endif**
∧ homogeneous1 (n, cdr ($domain$), $pairs$, flg)
else t endif

DEFINITION:
homogeneous ($domain$, $pairs$, flg)
= **if** listp ($domain$)
then homogeneous1 (car ($domain$), cdr ($domain$), $pairs$, flg)
∧ homogeneous (cdr ($domain$), $pairs$, flg)
else t endif

DEFINITION:
subsetp (x, y)
= **if** $x \simeq$ **nil then t**
elseif car (x) $\in y$ **then** subsetp (cdr (x), y)
else f endif

THEOREM: member-cons
$(a \in l) \rightarrow (a \in \text{cons}(x, l))$

THEOREM: subsetp-cons
$\text{subsetp}(l, m) \rightarrow \text{subsetp}(l, \text{cons}(a, m))$

THEOREM: subsetp-reflexivity
$\text{subsetp}(x, x)$

THEOREM: cdr-subsetp
$\text{subsetp}(\text{cdr}(x), x)$

THEOREM: member-subsetp
$((x \in y) \wedge \text{subsetp}(y, z)) \rightarrow (x \in z)$

THEOREM: subsetp-is-transitive
$(\text{subsetp}(x, y) \wedge \text{subsetp}(y, z)) \rightarrow \text{subsetp}(x, z)$

THEOREM: subsetp-cdr-partition
$\text{subsetp}(\text{cdr}(\text{partition}(x, z, pairs)), z)$

THEOREM: subsetp-hom-set-domain-1

$$\text{subsetp}(\text{car}(\text{wit}(\textit{pairs}, \text{cdr}(\text{partition}(x, z, \textit{pairs})), p, q)), \text{cdr}(\text{partition}(x, z, \textit{pairs}))) \rightarrow \text{subsetp}(\text{car}(\text{wit}(\textit{pairs}, \text{cdr}(\text{partition}(x, z, \textit{pairs})), p, q)), \text{cons}(x, z))$$

THEOREM: subsetp-car-partition

$$\text{subsetp}(\text{car}(\text{partition}(x, z, \textit{pairs})), z)$$

THEOREM: subsetp-hom-set-domain-2

$$\text{subsetp}(\text{car}(\text{wit}(\textit{pairs}, \text{car}(\text{partition}(x, z, \textit{pairs})), p, q)), \text{car}(\text{partition}(x, z, \textit{pairs}))) \rightarrow \text{subsetp}(\text{car}(\text{wit}(\textit{pairs}, \text{car}(\text{partition}(x, z, \textit{pairs})), p, q)), \text{cons}(x, z))$$

THEOREM: subsetp-hom-set-domain

$$\text{subsetp}(\text{car}(\text{wit}(\textit{pairs}, \textit{domain}, p, q)), \textit{domain})$$

DEFINITION:

$$\text{good-hom-set}(\textit{pairs}, \textit{domain}, p, q, \textit{flg}) = (\text{homogeneous}(\text{car}(\text{wit}(\textit{pairs}, \textit{domain}, p, q)), \textit{pairs}, \textit{flg}) \land (\text{length}(\text{car}(\text{wit}(\textit{pairs}, \textit{domain}, p, q))) \not< \textbf{if } \textit{flg} = \texttt{1} \textbf{ then } p \textbf{ else } q \textbf{ endif}))$$

THEOREM: homogeneous1-subset

$$(\text{subsetp}(x, \textit{domain}) \land \text{homogeneous1}(\textit{elt}, \textit{domain}, \textit{pairs}, \textit{flg})) \rightarrow \text{homogeneous1}(\textit{elt}, x, \textit{pairs}, \textit{flg})$$

THEOREM: homogeneous1-cdr-partition

$$\text{homogeneous1}(\textit{elt}, \text{cdr}(\text{partition}(\textit{elt}, \textit{dom}, \textit{pairs})), \textit{pairs}, \texttt{2})$$

THEOREM: length-partition-1

$$\text{length}(z) = (\text{length}(\text{car}(\text{partition}(x, z, \textit{pairs}))) + \text{length}(\text{cdr}(\text{partition}(x, z, \textit{pairs}))))$$

THEOREM: length-partition

$$\text{length}(\text{car}(\text{partition}(x, z, \textit{pairs}))) = (\text{length}(z) - \text{length}(\text{cdr}(\text{partition}(x, z, \textit{pairs}))))$$

THEOREM: ramsey-not-zero

$$\texttt{0} < \text{ramsey}(p, q)$$

THEOREM: homogeneous1-car-wit-cdr-partition

$$\text{homogeneous1}(\textit{elt}, \text{car}(\text{wit}(\textit{pairs}, \text{cdr}(\text{partition}(\textit{elt}, \textit{dom}, \textit{pairs})), p, q)), \textit{pairs}, \texttt{2})$$

THEOREM: homogeneous1-car-partition

$$\text{homogeneous1}(\textit{elt}, \text{car}(\text{partition}(\textit{elt}, \textit{dom}, \textit{pairs})), \textit{pairs}, \texttt{1})$$

THEOREM: homogeneous1-car-wit-car-partition
homogeneous1 (*elt*, car (wit (*pairs*, car (partition (*elt*, *dom*, *pairs*)), *p*, *q*)), *pairs*, 1)

THEOREM: cdr-wit-is-1-or-2
(cdr (wit (*pairs*, *dom*, *p*, *q*)) $\neq$ 1) $\rightarrow$ (cdr (wit (*pairs*, *dom*, *p*, *q*)) = 2)

THEOREM: wit-yields-good-hom-set
(length (*domain*) $\not<$ ramsey (*p*, *q*))
$\rightarrow$ good-hom-set (*pairs*, *domain*, *p*, *q*, cdr (wit (*pairs*, *domain*, *p*, *q*)))

DEFINITION:
setp (*x*)
= **if** listp (*x*) **then** (car (*x*) $\notin$ cdr (*x*)) $\wedge$ setp (cdr (*x*))
else t endif

THEOREM: setp-partition
setp (*x*)
$\rightarrow$ (setp (car (partition (*a*, *x*, *pairs*))) $\wedge$ setp (cdr (partition (*a*, *x*, *pairs*))))

THEOREM: not-member-car-wit-cdr-partition
(*z* $\notin$ *x*) $\rightarrow$ (*z* $\notin$ car (wit (*pairs*, cdr (partition (*z*, *x*, *pairs*)), *p*, *q*)))

THEOREM: not-member-car-wit-car-partition
(*z* $\notin$ *x*) $\rightarrow$ (*z* $\notin$ car (wit (*pairs*, car (partition (*z*, *x*, *pairs*)), *p*, *q*)))

THEOREM: setp-hom-set
setp (*domain*) $\rightarrow$ setp (car (wit (*pairs*, *domain*, *p*, *q*)))

THEOREM: ramsey-theorem-2
(ramsey (*p*, *q*) $\leq$ length (*domain*))
$\rightarrow$ (subsetp (car (wit (*pairs*, *domain*, *p*, *q*)), *domain*)
$\wedge$ good-hom-set (*pairs*, *domain*, *p*, *q*, cdr (wit (*pairs*, *domain*, *p*, *q*)))
$\wedge$ (setp (*domain*) $\rightarrow$ setp (car (wit (*pairs*, *domain*, *p*, *q*)))))

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