

#|

Copyright (C) 1994 by Computational Logic, Inc. All Rights Reserved.

This script is hereby placed in the public domain, and therefore unlimited editing and redistribution is permitted.

NO WARRANTY

Computational Logic, Inc. PROVIDES ABSOLUTELY NO WARRANTY. THE EVENT SCRIPT IS PROVIDED "AS IS" WITHOUT WARRANTY OF ANY KIND, EITHER EXPRESS OR IMPLIED, INCLUDING, BUT NOT LIMITED TO, ANY IMPLIED WARRANTIES OF MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE. THE ENTIRE RISK AS TO THE QUALITY AND PERFORMANCE OF THE SCRIPT IS WITH YOU. SHOULD THE SCRIPT PROVE DEFECTIVE, YOU ASSUME THE COST OF ALL NECESSARY SERVICING, REPAIR OR CORRECTION.

IN NO EVENT WILL Computational Logic, Inc. BE LIABLE TO YOU FOR ANY DAMAGES, ANY LOST PROFITS, LOST MONIES, OR OTHER SPECIAL, INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF THE USE OR INABILITY TO USE THIS SCRIPT (INCLUDING BUT NOT LIMITED TO LOSS OF DATA OR DATA BEING RENDERED INACCURATE OR LOSSES SUSTAINED BY THIRD PARTIES), EVEN IF YOU HAVE ADVISED US OF THE POSSIBILITY OF SUCH DAMAGES, OR FOR ANY CLAIM BY ANY OTHER PARTY.

|#

EVENT: Start with the initial **nqthm** theory.

DEFINITION:

ramsey(p, q)

= **if** $p \simeq 0$ **then** 1
 elseif $q \simeq 0$ **then** 1
 else ramsey($p - 1, q$) + ramsey($p, q - 1$) **endif**

DEFINITION:

related($i, j, pairs$) = ((cons(i, j) \in $pairs$) \vee (cons(j, i) \in $pairs$))

DEFINITION:

partition($n, rest, pairs$)

= **if** listp($rest$)
 then if related($n, car(rest), pairs$)
 then cons(cons(car($rest$), car(partition($n, cdr(rest), pairs$))),
 cdr(partition($n, cdr(rest), pairs$)))
 else cons(car(partition($n, cdr(rest), pairs$)),
 cons(car($rest$), cdr(partition($n, cdr(rest), pairs$)))) **endif**
 else cons(**nil**, **nil**) **endif**

DEFINITION:

```
length(lst)
=  if listp(lst) then 1 + length(cdr(lst))
  else 0 endif
```

DEFINITION:

```
wit(pairs, domain, p, q)
=  if listp(domain)
    then if  $p \simeq 0$  then cons(nil, 1)
          elseif  $q \simeq 0$  then cons(nil, 2)
          elseif length(car(partition(car(domain), cdr(domain), pairs)))
                   < ramsey( $p - 1$ ,  $q$ )
          then if cdr(wit(pairs,
                          cdr(partition(car(domain), cdr(domain), pairs)),
                          p,
                           $q - 1$ ))
              = 1
          then wit(pairs,
                  cdr(partition(car(domain), cdr(domain), pairs)),
                  p,
                   $q - 1$ )
          else cons(cons(car(domain),
                          car(wit(pairs,
                                  cdr(partition(car(domain),
                                                  cdr(domain),
                                                  pairs)),
                                  p,
                                   $q - 1$ ))),
                    2) endif
          elseif cdr(wit(pairs,
                          car(partition(car(domain), cdr(domain), pairs)),
                           $p - 1$ ,
                           $q$ ))
              = 2
          then wit(pairs,
                  car(partition(car(domain), cdr(domain), pairs)),
                   $p - 1$ ,
                   $q$ )
          else cons(cons(car(domain),
                          car(wit(pairs,
                                  car(partition(car(domain),
                                                  cdr(domain),
                                                  pairs)),
                                   $p - 1$ ,
                                   $q$ ))),
                     $p - 1$ ,
```

```

1) endif
else cons(nil, 1) endif

```

DEFINITION:

```

homogeneous1(n, domain, pairs, flg)
=  if listp(domain)
    then if flg = 1 then related(n, car(domain), pairs)
        else ¬ related(n, car(domain), pairs) endif
        ∧ homogeneous1(n, cdr(domain), pairs, flg)
    else t endif

```

DEFINITION:

```

homogeneous(domain, pairs, flg)
=  if listp(domain)
    then homogeneous1(car(domain), cdr(domain), pairs, flg)
         $\wedge$  homogeneous(cdr(domain), pairs, flg)
    else t endif

```

DEFINITION:

```

subsetp(x, y)
=  if x  $\simeq$  nil then t
    elseif car(x)  $\in$  y then subsetp(cdr(x), y)
    else f endif

```

THEOREM: member-cons
 $(a \in l) \rightarrow (a \in \text{cons}(x, l))$

THEOREM: subsetp-cons
 $\text{subsetp}(l, m) \rightarrow \text{subsetp}(l, \text{cons}(a, m))$

THEOREM: subsetp-reflexivity
 $\text{subsetp}(x, x)$

THEOREM: cdr-subsetp
 $\text{subsetp}(\text{cdr}(x), x)$

THEOREM: member-subsetp
 $((x \in y) \wedge \text{subsetp}(y, z)) \rightarrow (x \in z)$

THEOREM: subsetp-is-transitive
 $(\text{subsetp}(x, y) \wedge \text{subsetp}(y, z)) \rightarrow \text{subsetp}(x, z)$

THEOREM: subsetp-cdr-partition
 $\text{subsetp}(\text{cdr}(\text{partition}(x, z, \text{pairs})), z)$

THEOREM: subsetp-hom-set-domain-1
 $\text{subsetp}(\text{car}(\text{wit}(\text{pairs}, \text{cdr}(\text{partition}(x, z, \text{pairs})), p, q)),$
 $\text{cdr}(\text{partition}(x, z, \text{pairs})))$
 $\rightarrow \text{subsetp}(\text{car}(\text{wit}(\text{pairs}, \text{cdr}(\text{partition}(x, z, \text{pairs})), p, q)), \text{cons}(x, z))$

THEOREM: subsetp-car-partition
 $\text{subsetp}(\text{car}(\text{partition}(x, z, \text{pairs})), z)$

THEOREM: subsetp-hom-set-domain-2
 $\text{subsetp}(\text{car}(\text{wit}(\text{pairs}, \text{car}(\text{partition}(x, z, \text{pairs})), p, q)),$
 $\text{car}(\text{partition}(x, z, \text{pairs})))$
 $\rightarrow \text{subsetp}(\text{car}(\text{wit}(\text{pairs}, \text{car}(\text{partition}(x, z, \text{pairs})), p, q)), \text{cons}(x, z))$

THEOREM: subsetp-hom-set-domain
 $\text{subsetp}(\text{car}(\text{wit}(\text{pairs}, \text{domain}, p, q)), \text{domain})$

DEFINITION:
 $\text{good-hom-set}(\text{pairs}, \text{domain}, p, q, \text{flag})$
 $= (\text{homogeneous}(\text{car}(\text{wit}(\text{pairs}, \text{domain}, p, q)), \text{pairs}, \text{flag})$
 $\wedge (\text{length}(\text{car}(\text{wit}(\text{pairs}, \text{domain}, p, q)))$
 $\not\prec \text{if } \text{flag} = 1 \text{ then } p$
 $\text{else } q \text{ endif}))$

THEOREM: homogeneous1-subset
 $(\text{subsetp}(x, \text{domain}) \wedge \text{homogeneous1}(\text{elt}, \text{domain}, \text{pairs}, \text{flag}))$
 $\rightarrow \text{homogeneous1}(\text{elt}, x, \text{pairs}, \text{flag})$

THEOREM: homogeneous1-cdr-partition
 $\text{homogeneous1}(\text{elt}, \text{cdr}(\text{partition}(\text{elt}, \text{dom}, \text{pairs})), \text{pairs}, 2)$

THEOREM: length-partition-1
 $\text{length}(z)$
 $= (\text{length}(\text{car}(\text{partition}(x, z, \text{pairs})))$
 $+ \text{length}(\text{cdr}(\text{partition}(x, z, \text{pairs}))))$

THEOREM: length-partition
 $\text{length}(\text{car}(\text{partition}(x, z, \text{pairs})))$
 $= (\text{length}(z) - \text{length}(\text{cdr}(\text{partition}(x, z, \text{pairs}))))$

THEOREM: ramsey-not-zero
 $0 < \text{ramsey}(p, q)$

THEOREM: homogeneous1-car-wit-cdr-partition
 $\text{homogeneous1}(\text{elt}, \text{car}(\text{wit}(\text{pairs}, \text{cdr}(\text{partition}(\text{elt}, \text{dom}, \text{pairs}))), p, q), \text{pairs}, 2)$

THEOREM: homogeneous1-car-partition
 $\text{homogeneous1}(\text{elt}, \text{car}(\text{partition}(\text{elt}, \text{dom}, \text{pairs})), \text{pairs}, 1)$

THEOREM: homogeneous1-car-wit-car-partition
 $\text{homogeneous1}(\text{elt}, \text{car}(\text{wit}(\text{pairs}, \text{car}(\text{partition}(\text{elt}, \text{dom}, \text{pairs}))), p, q), \text{pairs}, 1)$

THEOREM: cdr-wit-is-1-or-2
 $(\text{cdr}(\text{wit}(\text{pairs}, \text{dom}, p, q)) \neq 1) \rightarrow (\text{cdr}(\text{wit}(\text{pairs}, \text{dom}, p, q)) = 2)$

THEOREM: wit-yields-good-hom-set
 $(\text{length}(\text{domain}) \not\leq \text{ramsey}(p, q))$
 $\rightarrow \text{good-hom-set}(\text{pairs}, \text{domain}, p, q, \text{cdr}(\text{wit}(\text{pairs}, \text{domain}, p, q)))$

DEFINITION:
 $\text{setp}(x)$
 $= \text{if listp}(x) \text{ then } (\text{car}(x) \notin \text{cdr}(x)) \wedge \text{setp}(\text{cdr}(x))$
 else t endif

THEOREM: setp-partition
 $\text{setp}(x)$
 $\rightarrow (\text{setp}(\text{car}(\text{partition}(a, x, \text{pairs}))) \wedge \text{setp}(\text{cdr}(\text{partition}(a, x, \text{pairs}))))$

THEOREM: not-member-car-wit-cdr-partition
 $(z \notin x) \rightarrow (z \notin \text{car}(\text{wit}(\text{pairs}, \text{cdr}(\text{partition}(z, x, \text{pairs})), p, q)))$

THEOREM: not-member-car-wit-car-partition
 $(z \notin x) \rightarrow (z \notin \text{car}(\text{wit}(\text{pairs}, \text{car}(\text{partition}(z, x, \text{pairs})), p, q)))$

THEOREM: setp-hom-set
 $\text{setp}(\text{domain}) \rightarrow \text{setp}(\text{car}(\text{wit}(\text{pairs}, \text{domain}, p, q)))$

THEOREM: ramsey-theorem-2
 $(\text{ramsey}(p, q) \leq \text{length}(\text{domain}))$
 $\rightarrow (\text{subsetp}(\text{car}(\text{wit}(\text{pairs}, \text{domain}, p, q)), \text{domain})$
 $\wedge \text{good-hom-set}(\text{pairs}, \text{domain}, p, q, \text{cdr}(\text{wit}(\text{pairs}, \text{domain}, p, q)))$
 $\wedge (\text{setp}(\text{domain}) \rightarrow \text{setp}(\text{car}(\text{wit}(\text{pairs}, \text{domain}, p, q))))$

Index

cdr-subsetp, 3
cdr-wit-is-1-or-2, 5

good-hom-set, 4, 5

homogeneous, 3, 4
homogeneous1, 3–5
homogeneous1-car-partition, 4
homogeneous1-car-wit-car-partition, 5
homogeneous1-car-wit-cdr-partition, 4
homogeneous1-cdr-partition, 4
homogeneous1-subset, 4

length, 2, 4, 5
length-partition, 4
length-partition-1, 4

member-cons, 3
member-subsetp, 3

not-member-car-wit-car-partition, 5
not-member-car-wit-cdr-partition, 5

partition, 1–5

ramsey, 1, 2, 4, 5
ramsey-not-zero, 4
ramsey-theorem-2, 5
related, 1, 3

setp, 5
setp-hom-set, 5
setp-partition, 5
subsetp, 3–5
subsetp-car-partition, 4
subsetp-cdr-partition, 3
subsetp-cons, 3
subsetp-hom-set-domain, 4
subsetp-hom-set-domain-1, 4
subsetp-hom-set-domain-2, 4
subsetp-is-transitive, 3
subsetp-reflexivity, 3

wit, 2–5
wit-yields-good-hom-set, 5