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; Events from CLI Technical Report 100, ``Interaction with the Boyer-Moore
; Theorem Prover: A Tutorial Study Using the Arithmetic-Geometric Mean
; Theorem,'' by Matt Kaufmann and Paolo Pecchiari.
```

EVENT: Start with the library `"naturals"` using the compiled version.

DEFINITION:
prodlist (lst)
= **if** listp (lst) **then** car (lst) $*$ prodlist (cdr (lst))
else `1` **endif**

DEFINITION:
sumlist (lst)
= **if** listp (lst) **then** car (lst) $+$ sumlist (cdr (lst))
else `0` **endif**

DEFINITION:
length (x)

= **if** listp (*x*) **then** 1 + length (cdr (*x*))
else 0 **endif**

DEFINITION:
maxlist (*x*)
= **if** listp (*x*)
then if listp (cdr (*x*)) **then** max (car (*x*), maxlist (cdr (*x*)))
else fix (car (*x*)) **endif**
else 0 **endif**

DEFINITION:
min (*x*, *y*)
= **if** *x* < *y* **then** fix (*x*)
else fix (*y*) **endif**

DEFINITION:
minlist (*x*)
= **if** listp (*x*)
then if listp (cdr (*x*)) **then** min (car (*x*), minlist (cdr (*x*)))
else fix (car (*x*)) **endif**
else 0 **endif**

DEFINITION:
delete1 (*elt*, *x*)
= **if** listp (*x*)
then if *elt* = car (*x*) **then** cdr (*x*)
else cons (car (*x*), delete1 (*elt*, cdr (*x*))) **endif**
else *x* **endif**

THEOREM: maxlist-delete1-rearrange
(*b* ∈ *x*) → (maxlist (cons (*b*, delete1 (*b*, *x*))) = maxlist (*x*))

EVENT: Let us define the theory *induction-fn-disables* to consist of the following events: sumlist, times, length, minlist, maxlist, delete1, occurrences.

THEOREM: max-greater-than-average
(listp (*x*) ∧ (sumlist (*x*) ≥ (*k* ∗ length (*x*))) ∧ (fix (*k*) ≠ maxlist (*x*)))
→ (*k* < maxlist (*x*))

THEOREM: min-less-than-average
(listp (*x*) ∧ (sumlist (*x*) ≤ (*k* ∗ length (*x*))) ∧ (fix (*k*) ≠ minlist (*x*)))
→ (minlist (*x*) < *k*)

THEOREM: times-monotone-1
(*u* ≮ *v*) → ((*u* ∗ *y*) ≮ (*v* ∗ *y*))

THEOREM: lessp-times-preserved-in-first-arg
$((a \not< (u * y)) \wedge (u \not< v)) \rightarrow (a \not< (v * y))$

THEOREM: minlist-main-property
$\text{sumlist}(x) \not< (\text{length}(x) * \text{minlist}(x))$

THEOREM: minlist-not-maxlist-implies-minlist-lessp-average-lemma
$(\text{minlist}(x) \neq \text{maxlist}(x)) \rightarrow ((\text{minlist}(x) * \text{length}(x)) < \text{sumlist}(x))$

THEOREM: minlist-not-maxlist-implies-minlist-lessp-average
$(\text{listp}(x)$
$\wedge \quad (\text{sumlist}(x) = (k * \text{length}(x)))$
$\wedge \quad (\text{minlist}(x) \neq \text{maxlist}(x)))$
$\rightarrow \quad (\text{minlist}(x) < \text{fix}(k))$

THEOREM: maxlist-main-property
$(\text{length}(x) * \text{maxlist}(x)) \not< \text{sumlist}(x)$

THEOREM: minlist-not-maxlist-implies-maxlist-greaterp-average-lemma
$(\text{minlist}(x) \neq \text{maxlist}(x)) \rightarrow (\text{sumlist}(x) < (\text{maxlist}(x) * \text{length}(x)))$

THEOREM: minlist-not-maxlist-implies-maxlist-greaterp-average
$(\text{listp}(x)$
$\wedge \quad (\text{sumlist}(x) = (k * \text{length}(x)))$
$\wedge \quad (\text{minlist}(x) \neq \text{maxlist}(x)))$
$\rightarrow \quad (\text{fix}(k) < \text{maxlist}(x))$

THEOREM: minlist-less-than-maxlist-minus-1
$(\text{listp}(x)$
$\wedge \quad (\text{sumlist}(x) = (k * \text{length}(x)))$
$\wedge \quad (\text{minlist}(x) \neq \text{maxlist}(x)))$
$\rightarrow \quad (\text{minlist}(x) < (\text{maxlist}(x) - 1))$

THEOREM: maxlist-delete1-leq
$\text{maxlist}(lst) \not< \text{maxlist}(\text{delete1}(a, lst))$

THEOREM: member-implies-maxlist-geq
$(a \in x) \rightarrow (\text{maxlist}(x) \not< a)$

THEOREM: delete1-preserves-maxlist-when-maxlist-occurs-more-than-once
$(1 < \text{occurrences}(\text{maxlist}(x), x))$
$\rightarrow \quad (\text{maxlist}(\text{delete1}(\textit{any-element}, x)) = \text{maxlist}(x))$

THEOREM: delete1-occurrences
$\text{occurrences}(a, \text{delete1}(b, x))$
$= \quad \textbf{if } (a = b) \wedge (b \in x) \textbf{ then } \text{occurrences}(a, x) - 1$
$\quad \textbf{else } \text{occurrences}(a, x) \textbf{ endif}$

THEOREM: occurrence-implies-listp
$(1 < \text{occurrences}\,(a,\ x)) \rightarrow (\text{listp}\,(\text{delete1}\,(a,\ x)) = \mathbf{t})$

THEOREM: maxlist-geq-minlist
$\text{maxlist}\,(x) \not< \text{minlist}\,(x)$

THEOREM: induction-fn-help-2-max-occurs-twice-lemma-1
$$\begin{array}{ll}
((\text{sumlist}\,(x) = (k * \text{length}\,(x))) & \\
\wedge & (\text{minlist}\,(x) \neq \text{maxlist}\,(x)) \\
\wedge & (1 < \text{occurrences}\,(\text{maxlist}\,(x),\ x))) \\
\rightarrow & (\text{maxlist}\,(\text{cons}\,(\text{maxlist}\,(x) - 1, \\
 & \qquad \text{cons}\,(1 + \text{minlist}\,(x), \\
 & \qquad\quad \text{delete1}\,(\text{maxlist}\,(x),\ \text{delete1}\,(\text{minlist}\,(x),\ x))))) \\
 & = \quad \text{maxlist}\,(x))
\end{array}$$

THEOREM: member-delete1
$$\begin{array}{ll}
(a \in \text{delete1}\,(b,\ c)) & \\
= & \textbf{if } a = b \textbf{ then } 1 < \text{occurrences}\,(b,\ c) \\
 & \textbf{else } a \in c \textbf{ endif}
\end{array}$$

THEOREM: member-implies-listp
$(a \in x) \rightarrow \text{listp}\,(x)$

THEOREM: induction-fn-help-2-max-occurs-twice-lemma-2
$$\begin{array}{ll}
((\text{sumlist}\,(x) = (k * \text{length}\,(x))) & \\
\wedge & (\text{minlist}\,(x) \neq \text{maxlist}\,(x)) \\
\wedge & (1 < \text{occurrences}\,(\text{maxlist}\,(x),\ x))) \\
\rightarrow & ((\text{occurrences}\,(\text{maxlist}\,(x), \\
 & \qquad \text{cons}\,(\text{maxlist}\,(x) - 1, \\
 & \qquad\quad \text{cons}\,(1 + \text{minlist}\,(x), \\
 & \qquad\qquad \text{delete1}\,(\text{maxlist}\,(x),\ \text{delete1}\,(\text{minlist}\,(x),\ x))))) \\
 & < \quad \text{occurrences}\,(\text{maxlist}\,(x),\ x)) \\
 & = \quad \mathbf{t})
\end{array}$$

EVENT: Disable member-implies-listp.

THEOREM: induction-fn-help-2-max-occurs-twice
$$\begin{array}{l}
\textbf{let } x0 \textbf{ be } \text{cons}\,(\text{maxlist}\,(x) - 1, \\
\qquad\qquad \text{cons}\,(1 + \text{minlist}\,(x), \\
\qquad\qquad\quad \text{delete1}\,(\text{maxlist}\,(x),\ \text{delete1}\,(\text{minlist}\,(x),\ x)))) \\
\textbf{in} \\
\begin{array}{ll}
((\text{sumlist}\,(x) = (k * \text{length}\,(x))) & \\
\wedge & (\text{minlist}\,(x) \neq \text{maxlist}\,(x)) \\
\wedge & (1 < \text{occurrences}\,(\text{maxlist}\,(x),\ x))) \\
\rightarrow & \text{ord-lessp}\,(\text{cons}\,(1 + \text{maxlist}\,(x0),\ \text{occurrences}\,(\text{maxlist}\,(x0),\ x0)), \\
 & \qquad \text{cons}\,(1 + \text{maxlist}\,(x),\ \text{occurrences}\,(\text{maxlist}\,(x),\ x))) \textbf{ endlet}
\end{array}
\end{array}$$

THEOREM: maxlist-not-minlist-implies-listp
$(\text{minlist}\,(x) \neq \text{maxlist}\,(x)) \rightarrow \text{listp}\,(x)$

THEOREM: minlist-less-than-maxlist-minus-1-better
$((\text{sumlist}\,(x) = (k * \text{length}\,(x))) \wedge (\text{minlist}\,(x) \neq \text{maxlist}\,(x)))$
$\rightarrow \quad (\text{minlist}\,(x) < (\text{maxlist}\,(x) - 1))$

EVENT: Disable minlist-less-than-maxlist-minus-1.

THEOREM: maxlist-delete1-delete1
$\text{maxlist}\,(\text{delete1}\,(b,\, x)) \not< \text{maxlist}\,(\text{delete1}\,(b,\, \text{delete1}\,(a,\, x)))$

THEOREM: member-maxlist
$(\text{maxlist}\,(z) \neq \mathtt{0}) \rightarrow (\text{maxlist}\,(z) \in z)$

THEOREM: lessp-maxlist-delete1-maxlist
$((\mathtt{0} < \text{maxlist}\,(x)) \wedge (\mathtt{1} \not< \text{occurrences}\,(\text{maxlist}\,(x),\, x)))$
$\rightarrow \quad (\text{maxlist}\,(\text{delete1}\,(\text{maxlist}\,(x),\, x)) < \text{maxlist}\,(x))$

THEOREM: induction-fn-help-2-max-occurs-once-main-lemma
$((\text{sumlist}\,(x) = (k * \text{length}\,(x)))$
$\wedge \quad (\text{minlist}\,(x) \neq \text{maxlist}\,(x))$
$\wedge \quad (\mathtt{1} \not< \text{occurrences}\,(\text{maxlist}\,(x),\, x)))$
$\rightarrow \quad ((\text{maxlist}\,(\text{cons}\,(\text{maxlist}\,(x) - 1,$
$\text{cons}\,(1 + \text{minlist}\,(x),$
$\text{delete1}\,(\text{maxlist}\,(x),\, \text{delete1}\,(\text{minlist}\,(x),\, x)))))$
$< \quad \text{maxlist}\,(x))$
$= \quad \mathbf{t})$

THEOREM: induction-fn-help-2-max-occurs-once
let *x0* **be** $\text{cons}\,(\text{maxlist}\,(x) - 1,$
$\text{cons}\,(1 + \text{minlist}\,(x),$
$\text{delete1}\,(\text{maxlist}\,(x),\, \text{delete1}\,(\text{minlist}\,(x),\, x))))$
in
$((\text{sumlist}\,(x) = (k * \text{length}\,(x)))$
$\wedge \quad (\text{minlist}\,(x) \neq \text{maxlist}\,(x))$
$\wedge \quad (\mathtt{1} \not< \text{occurrences}\,(\text{maxlist}\,(x),\, x)))$
$\rightarrow \quad \text{ord-lessp}\,(\text{cons}\,(1 + \text{maxlist}\,(x0),\, \text{occurrences}\,(\text{maxlist}\,(x0),\, x0)),$
$\text{cons}\,(1 + \text{maxlist}\,(x),\, \text{occurrences}\,(\text{maxlist}\,(x),\, x)))$ **endlet**

THEOREM: induction-fn-help-2
let *x0* **be** $\text{cons}\,(\text{maxlist}\,(x) - 1,$
$\text{cons}\,(1 + \text{minlist}\,(x),$
$\text{delete1}\,(\text{maxlist}\,(x),\, \text{delete1}\,(\text{minlist}\,(x),\, x))))$
in

((sumlist (*x*) = (*k* ∗ length (*x*))) ∧ (minlist (*x*) ≠ maxlist (*x*)))
→ ord-lessp (cons (1 + maxlist (*x0*), occurrences (maxlist (*x0*), *x0*)),
cons (1 + maxlist (*x*), occurrences (maxlist (*x*), *x*))) **endlet**

EVENT: Disable theory induction-fn-disables.

DEFINITION:
induction-fn (*x*, *k*)
= **if** sumlist (*x*) ≠ (*k* ∗ length (*x*)) **then t**
elseif minlist (*x*) = maxlist (*x*) **then t**
else induction-fn (cons (maxlist (*x*) − 1,
cons (1 + minlist (*x*),
delete1 (maxlist (*x*),
delete1 (minlist (*x*), *x*)))),
k) **endif**

EVENT: Enable theory induction-fn-disablesinduction-fn-disables.

THEOREM: sumlist-delete1-plus-version
(*a* + sumlist (delete1 (*a*, *x*)))
= **if** *a* ∈ *x* **then** sumlist (*x*)
else *a* + sumlist (*x*) **endif**

THEOREM: sumlist-delete1
sumlist (delete1 (*a*, *x*))
= **if** *a* ∈ *x* **then** sumlist (*x*) − *a*
else sumlist (*x*) **endif**

THEOREM: maxlist-0-is-sumlist-0
(maxlist (*x*) = 0) = (sumlist (*x*) = 0)

THEOREM: main-lemma-base-case-lemma-1
(minlist (*x*) = maxlist (*x*)) → (exp (minlist (*x*), length (*x*)) = prodlist (*x*))

THEOREM: main-lemma-base-case-lemma-2-lemma
(minlist (*x*) = maxlist (*x*)) → (sumlist (*x*) = (minlist (*x*) ∗ length (*x*)))

THEOREM: main-lemma-base-case-lemma-2-hack
(((*k* ∗ *n*) = *sumlist*) ∧ (*n* ≠ 0) ∧ ((*n* ∗ *maxlist*) = *sumlist*))
→ (fix (*k*) = fix (*maxlist*))

THEOREM: equal-length-0
(length (*x*) = 0) = (¬ listp (*x*))

THEOREM: main-lemma-base-case-lemma-2
$(\text{listp}\,(x)$
$\wedge \quad (\text{sumlist}\,(x) = (k * \text{length}\,(x)))$
$\wedge \quad (\text{minlist}\,(x) = \text{maxlist}\,(x)))$
$\rightarrow \quad (\text{minlist}\,(x) = \text{fix}\,(k))$

THEOREM: main-lemma-base-case
$((\text{sumlist}\,(x) = (k * \text{length}\,(x))) \wedge (\text{minlist}\,(x) = \text{maxlist}\,(x)))$
$\rightarrow \quad (\exp\,(k, \text{length}\,(x)) = \text{prodlist}\,(x))$

THEOREM: length-delete1
$\text{length}\,(\text{delete1}\,(a, x))$
$= \quad \textbf{if } a \in x \textbf{ then } \text{length}\,(x) - 1$
$\quad\quad \textbf{else } \text{length}\,(x) \textbf{ endif}$

DEFINITION:
$\text{numberp-listp}\,(x)$
$= \quad \textbf{if } \text{listp}\,(x) \textbf{ then } (\text{car}\,(x) \in \mathbf{N}) \wedge \text{numberp-listp}\,(\text{cdr}\,(x))$
$\quad\quad \textbf{else } x = \textbf{nil endif}$

THEOREM: member-minlist
$(\text{listp}\,(x) \wedge \text{numberp-listp}\,(x)) \rightarrow (\text{minlist}\,(x) \in x)$

THEOREM: numberp-listp-delete1
$\text{numberp-listp}\,(x) \rightarrow \text{numberp-listp}\,(\text{delete1}\,(a, x))$

THEOREM: add1-plus-sub1-second
$(y \not\simeq 0) \rightarrow ((1 + (x + (y - 1))) = (x + y))$

THEOREM: sumlist-geq-maxlist
$\text{sumlist}\,(x) \not< \text{maxlist}\,(x)$

THEOREM: sumlist-geq-minlist
$\text{sumlist}\,(x) \not< \text{minlist}\,(x)$

THEOREM: sumlist-geq-minlist-plus-maxlist
$(\text{minlist}\,(x) \neq \text{maxlist}\,(x))$
$\rightarrow \quad (\text{sumlist}\,(x) \not< (\text{minlist}\,(x) + \text{maxlist}\,(x)))$

EVENT: Disable plus.

THEOREM: plus-times-sub1-second
$(y \not\simeq 0) \rightarrow ((x + (x * (y - 1))) = (x * y))$

EVENT: Disable times.

THEOREM: minlist-not-maxlist-implies-length-at-least-2
$(\text{minlist}\,(x) \neq \text{maxlist}\,(x)) \rightarrow (1 < \text{length}\,(x))$

EVENT: Disable plus-add1-arg1.

EVENT: Disable plus-add1-arg2.

EVENT: Disable times-add1.

EVENT: Disable maxlist-0-is-sumlist-0.

THEOREM: times-prodlist-delete1
$(a \in x) \rightarrow ((a * \text{prodlist}\,(\text{delete1}\,(a,\, x))) = \text{prodlist}\,(x))$

THEOREM: product-of-modified-list-lemma
$((min \neq max) \wedge (min \in x) \wedge (max \in x))$
$\rightarrow \quad ((max * min * \text{prodlist}\,(\text{delete1}\,(max,\, \text{delete1}\,(min,\, x))))$
$\quad = \quad \text{prodlist}\,(x))$

EVENT: Disable times-prodlist-delete1.

THEOREM: product-of-modified-list-lemma-2
$((min < max) \wedge (max \not\simeq 0))$
$\rightarrow \quad (((1 + min) * (max - 1) * rest)$
$\quad = \quad ((min * max * rest) + ((max - (1 + min)) * rest)))$

THEOREM: positive-implies-numberp
$(min < max) \rightarrow (max \in \mathbf{N})$

THEOREM: product-of-modified-list
$((min < max) \wedge (min \in x) \wedge (max \in x))$
$\rightarrow \quad$ **let** *rest* **be** $\text{prodlist}\,(\text{delete1}\,(max,\, \text{delete1}\,(min,\, x)))$
in
$((1 + min) * (max - 1) * rest)$
$= \quad (\text{prodlist}\,(x) + ((max - (1 + min)) * rest))$ **endlet**

THEOREM: main-lemma
let n **be** $\text{length}\,(x)$
in
$(\text{numberp-listp}\,(x) \wedge (\text{sumlist}\,(x) = (k * n)))$
$\rightarrow \quad (\exp\,(k,\, n) \not< \text{prodlist}\,(x))$ **endlet**

DEFINITION:
scalar-product (*scalar*, *lst*)
= **if** listp (*lst*)
then cons (*scalar* ∗ car (*lst*), scalar-product (*scalar*, cdr (*lst*)))
else nil endif

THEOREM: sumlist-scalar-product
sumlist (scalar-product (*scalar*, *lst*)) = (*scalar* ∗ sumlist (*lst*))

THEOREM: prodlist-scalar-product
prodlist (scalar-product (*scalar*, *lst*))
= (exp (*scalar*, length (*lst*)) ∗ prodlist (*lst*))

THEOREM: length-scalar-product
length (scalar-product (*n*, *a*)) = length (*a*)

THEOREM: numberp-listp-scalar-product
numberp-listp (*a*) → numberp-listp (scalar-product (*n*, *a*))

THEOREM: main
numberp-listp (*a*)
→ **let** *n* **be** length (*a*)
in
exp (sumlist (*a*), *n*) ≥ (exp (*n*, *n*) ∗ prodlist (*a*)) **endlet**

THEOREM: sumlist-for
sumlist (*a*)
= **for** *x* **in** *a*
sum *x* **endfor**

THEOREM: prodlist-for
prodlist (*a*)
= **for** *x* **in** *a*
multiply *x* **endfor**

THEOREM: main-again
(numberp-listp (*a*) ∧ (*n* = length (*a*)))
→ (exp (**for** *x* **in** *a*
sum *x* **endfor**,
n)
≥ (exp (*n*, *n*)
∗ **for** *x* **in** *a*
multiply *x* **endfor**))

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